A NOTE ON A THEOREM OF W. B. FORD

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W. B. Ford's theorem as stated in [1] on page 205 is incorrect. We observe that in Ford's proof, he claims

$$\lim_{n\to\infty} D_n = 0$$

on page 207 in [1]. But his hypotheses do not guarantee at all that $D_n \to 0$ as $n \to \infty$, when

$$\max_{n\to\infty} |g(2n + \tfrac{1}{2} + iy)| = \infty$$

for small values of y. Ford's proof holds, if we make an accurate restatement of Ford's theorem with appropriate generality, as follows:

If the coefficient $g(n)$ of the power series

$$f(z) = \sum_{n=0}^{\infty} g(n)z^n \tag{1}$$

radius of convergence > 0 may be considered as a function $g(s)$ of the complex variable $s = x + iy$ and as such satisfies the following two conditions, when considered throughout each right half plane $x > x_0$, where x_0 is any arbitrary large negative number.

(a) The function $g(s)$ is single valued and analytic except for a finite number of poles situated at the points $s = s_1, s_2, \cdots, s_p$ which lies within a Band B:

$$|Im\, s_i| < c, \qquad Re\, s_i < c,$$

where c is a fixed positive constant and $i = 1, 2, \cdots, p$. Furthermore, none of the s_i is a negative integer and p may increase as x_0 is decreased.

(b) For any point $s = x + iy$ to the right of the line $x = x_0$ and outside the Band B,

$$|g(x+iy)| < ke^{(\gamma+\epsilon)|y|}, \tag{2}$$

where γ is some fixed value such that $0 < \gamma < \pi$ and ϵ is any positive number. The value of k depends upon x_0 and ϵ.

Then the function $g(s)$ as defined by (1) will be analytic in a sector $S: \gamma < \arg z < 2\pi - \gamma$ and for z's of large modulus in Sector S, $f(z)$ may be developed asymptotically

$$f(z) \approx \sum_{n=1}^{\infty} r_n - \sum_{n=1}^{\infty} \frac{g(-n)}{z^n}, \tag{3}$$

where r_n represents the residue of the function

$$\frac{\pi g(x)(-z)^s}{\sin \pi s}$$

at the point $s = s_n$, $n = 1, 2, \cdots, p$.

REFERENCE

1. W. B. Ford, *Asymptotic Series and Divergent Series,* Chelsea Publishing Company, New York, 1960.

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