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DIOPHANTINE REPRESENTATION OF THE LUCAS NUMBERS

JAMES P. JONES
University of Calgary, Calgary, Alberta, Canada

The Lucas numbers, 1, 3, 4, 7, 11, 18, 29, ⋯, are defined recursively by the equations

$$L_1 = 1, \qquad L_2 = 3 \qquad \text{and} \qquad L_{n+2} = L_{n+1} + L_n .$$

We shall show that the Lucas numbers may be defined by a particularly simple Diophantine equation and thus exhibit them as the positive numbers in the range of a very simple polynomial of the 9th degree.

Our results are based upon the following identity

$$L_{n+1}^2 - L_{n+1}L_n - L_n^2 = 5(-1)^{n+1} . \tag{1}$$

This identity (cf. [1] p. 2 No. 6) actually *defines* the Lucas numbers in the following sense.

Theorem 1. For any positive integer y, in order that y be a Lucas number, it is necessary and sufficient that there exist a positive number x such that

$$y^2 - yx - x^2 = \pm 5 . \tag{2}$$

Proof. The Proof is virtually identical to that of the analogous result for Fibonacci numbers proved in [2].

Theorem 2. The set of all Lucas numbers is identical with the position values of the polynomial

$$y(1 - ((y^2 - yx - x^2)^2 - 25)^2) \tag{3}$$

as the variables x and y range over the positive integers.

Proof. We have only to observe that the right factor of (3) cannot be positive unless equation (2) holds. Here we are using an idea of Putnam [3].

It will be seen that the polynomial (3) also gives certain negative values. This is unavoidable. It is easy to prove that a polynomial which takes *only* Lucas number values must be constant (cf. [2] Theorem 3).

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