## 172 DIOPHANTINE EQUATIONS INVOLVING THE GREATEST INTEGER FUNCTION [APR. 1977]

**Proof.** This follows from the Theorem with  $\mu = 1 + 1/\sigma$ ,  $\lambda = \sigma + 1$ , and b = c = 1. Q.E.D. (Corollary 3 is part of Problem 22 in [3, p. 84].)

## REFERENCES

- H. S. M. Coxeter, "The Golden Section, Phyllotaxis, and Wythoff's Game," Scripta Mathematica 19 (1953), pp. 135–143.
- 2. G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, 4th Ed., Oxford, 1960.
- 3. I. Niven and H. Zuckerman, An Introduction to the Theory of Numbers, 3rd ed., Wiley, N. Y., 1972.

### [Continued from page 149.]

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For an  $\omega$ -series with an arbitrary odd number of  $k_i$  parameters two cycles of parametric incrementation are required to bring the series into alignment for grouping. Use of the identity

$$G(z) = \psi(z/2 + 1/2) - \psi(z/2),$$

[4, p. 20], and Lemma 1 render the following summation expression. The orem 2.

$$\omega(j; k_1, \cdots, k_{2n+1}) = \sum_{i=0}^{2n} (-1)^i \omega(j + s_i; S) = (1/2S) \sum_{i=0}^{2n} (-1)^i G((j + s_i)/S).$$

# 3. EXAMPLES

Some calculations for the uniparameter  $\omega$ -series are to be found in [1] and for the biparameter series in [2]. The above theorems and their proofs can be illustrated with the following triparameter  $\omega$ -series:

$$\begin{split} \omega\left(1;\,1,\,1,\,2\right) &= \left[\left(1-1/2\right)+\left(1/3-1/5\right)+\left(1/6-1/7\right)\right]+\left[\left(1/9-1/10\right)+\left(1/11-1/13\right)+\dots\right] \\ &+ \left[\left(1/17-1/18\right)+\dots\right]+\dots \\ &= \left(1-1/2\right)+\left(1/9-1/10\right)+\left(1/17-1/18\right)+\dots+\left(1/3-1/5\right)+\left(1/11-1/13\right)+\dots \\ &+ \left(1/6-1/7\right)+\dots \\ &= \omega\left(1;\,1,\,7\right)+\omega\left(3;\,2,\,6\right)+\omega\left(6;\,1,\,7\right) \\ &= \left(1/8\right)\left[G\left(3/4\right)-G\left(1/2\right)+G\left(1/4\right)\right] \\ &= \left(1/8\right)\left[\sqrt{2}\left(\pi-2\ln\left(1+\sqrt{2}\right)-\pi+\sqrt{2}\left(\pi+2\ln\left(1+\sqrt{2}\right)\right)\right] \end{split}$$

$$= (\pi/8)[2\sqrt{2} - 1].$$

### REFERENCES

- B. J. Cerimele, "Extensions on a Theme Concerning Conditionally Convergent Series," *Mathematics Mag.*, Vol. 40, No. 3, May, 1967.
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- 3. H. T. Davis, "Tables of Higher Mathematical Functions," The Principia Press, 1933.
- 4. A. Erdelyi (ed), *Higher Transcendental Functions*, Vol. 1, McGraw Hill, 1953.
- 5. W. Grobner and N. Hofreiter, *Integraltafel*, Vol. 1, Springer-Verlag, 1961.
- 6. J. B. W. Jolly, Summation of Series, Dover Publications, 1961.

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