Proof. It can be verified that the polynomial defined in (1) has a generating function

$$(6)\qquad (t-d)^{(n)} = \sum_{m=0}^{n} t^m \alpha_{m+1,n+1}(d), \quad \text{where} \quad (t-d)^{(n)} = (t-d)(t-d-1)\cdots(t-d-n+1).$$

The generating function of $\beta_{m+1,n+1}(d)$ can be written

$$(7)\qquad t^n = \sum_{m=0}^{n} (t-d)^{(m)} \beta_{m+1,n+1}(d).$$

Using (6) and (7), (5) follows. This completes the proof of Theorem 2.

EXAMPLE: For $n = 3$, let

$$A = \begin{bmatrix} \alpha_{1,1}(d) & 0 & 0 & 0 \\ \alpha_{1,2}(d) & \alpha_{2,2}(d) & 0 & 0 \\ \alpha_{1,3}(d) & \alpha_{2,3}(d) & \alpha_{3,3}(d) & 0 \\ \alpha_{1,4}(d) & \alpha_{2,4}(d) & \alpha_{3,4}(d) & \alpha_{4,4}(d) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -d & 1 & 0 & 0 \\ d(d+1) & -(2d+1) & 1 & 0 \\ -d(d+1)(d+2) & (3d^2+6d+2) & -3(d+1) & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} \beta_{1,1}(d) & 0 & 0 & 0 \\ \beta_{1,2}(d) & \beta_{2,2}(d) & 0 & 0 \\ \beta_{1,3}(d) & \beta_{2,3}(d) & \beta_{3,3}(d) & 0 \\ \beta_{1,4}(d) & \beta_{2,4}(d) & \beta_{3,4}(d) & \beta_{4,4}(d) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ d & 1 & 0 & 0 \\ d^2 & (2d+1) & 1 & 0 \\ d^3 & (3d^2+3d+1) & 3(d+1) & 1 \end{bmatrix}$$

Then $A \cdot B = I$.

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PROBLEMS

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Proposed by Guy A. R. Guillot, Montreal, Quebec, Canada.

Prove that

$$\sum_{n=1}^{\infty} \tan^{-1} \frac{1}{n^2+n+1} = \sum_{n=1}^{\infty} \tan^{-1} \frac{1}{F_{2n+1}}.$$

Proposed by Guy A. R. Guillot, Montreal, Quebec, Canada.

Show that

$$\text{(a)}\qquad \sum_{n=1}^{\infty} \frac{1}{n^2 F_{n+2}} > \frac{\pi^2}{12} - \frac{(\log 2)^2}{2} + \frac{1}{48}$$

$$\text{(b)}\qquad \sum_{n=0}^{\infty} \frac{1}{F_{n+2}} \left(\tan \frac{\pi}{2^{n+2}}\right) > \frac{4}{\pi} + 0.0166.$$

[Continued on p. 257.]