MORE REDUCED AMICABLE PAIRS

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INTRODUCTION

Perfect, amicable and sociable numbers are the fixed points of the arithmetic function L and its iterates. $L(n) = \sigma(n) - n$, where σ is the sum of divisors function. Recently, there has been interest in fixed points of functions L_+, L_-, $L_\pm(n) = L(n) \pm 1$, and their iterates. Jerrard and Temperley [1] studied the fixed points of L_+ and L_-. Lal and Forbes [2] conducted a computer search for fixed points of $(L_-)^2$. For earlier references to L_-, see the bibliography in [2].

We conducted computer searches for fixed points n of iterates of L_- and L_+. Fixed points occur in sets where the number of elements in the set equals the power of L_- or L_+ in question.

In §1, we describe the results of L_-. The previous work of Lal and Forbes [2] discovered the fixed points of $(L_-)^2$ with one element of each pair $\leqslant 10^5$. We extend the results to $n \leqslant 10^6$. No other types of fixed points were discovered

The results for L_+ are described in §2. Again only pairs were found.

1. THE FUNCTION L_-

Lal and Forbes [2] discovered nine pairs of fixed points of $(L_-)^2$, where at least one element was less than, or equal to, 10^5. In fact, for all pairs, both numbers were less than 10^5.

If n is a fixed point of $(L_-)^k$: i.e. $(L_-)^k(n) = n$, for $k \geqslant 1$, then $(L_-)(n), (L_-)^2(n), \cdots, (L_-)^{k-1}(n)$ are also fixed points of $(L_-)^k$. Thus fixed points of iterates of L_- occur in sets of cardinality k. For at least one integer n in such a set, $L_-(n) > n$. Thus it suffices to search among n with $L_-(n) > n$.

A computer search was conducted using an IBM 370, Model 135. All natural numbers n, $0 < n \leqslant 10^6$, $L_-(n) > n$ were examined. The iterates $(L_-)^k(n)$, $1 \leqslant k \leqslant 50$, were calculated. Calculation of iterates stopped if

(a) $$(L_-)^m(n) = 0, \qquad 1 \leqslant m \leqslant 50;$$

or

(b) $$(L_-)^{m+k}(n) = (L_-)^m(n), \qquad 1 \leqslant k \leqslant 4.$$

The printout was to list all iterates calculated in case (b) and for the case where $(L_-)^{50}(n) > 0$. The program would discover any sets of fixed points arising from iterating L_- on integers n, $10^5 < n \leqslant 10^6$. We found six new pairs of reduced amicable numbers. There were no sets of fixed points of cardinality other than 2. Of the twelve numbers, only one exceeded 10^6. The pairs are listed in Table 1 with the prime factorization.

Table 1
L_-

(a)	186615	=	3(2)5·11·13·29
	206504	=	2(3)83·311
(b)	196664	=	2(3)13·31·61
	219975	=	3·5(2)7·419
(c)	199760	=	2(4)5·11·227
	309135	=	3·5·37·557
(d)	266000	=	2(4)5(3)7·19
	507759	=	3·7·24179
(e)	312620	=	2(2)5·7(2)11·29
	549219	=	3·11(2)17·89
(f)	587460	=	2(2)3·5·9791
	1057595	=	5·7·11·41·67

2. THE FUNCTION L_+

Jerrard and Temperley [2] ran a search for fixed points of L_+. Every power of 2 is a fixed point. But they discovered no others. They did not examine fixed points of iterates of L_+.

We call natural numbers *augmented perfect numbers, augmented amicable numbers* and *augmented sociable numbers* as they are fixed points of L_+, of $(L_+)^2$ or of $(L_+)^k$, $k > 2$. The names are suggested by the name *reduced amicable numbers* for fixed points of $(L_-)^2$ as used in [2].

A computer search for fixed points was run in the range, $0 < n \leqslant 10^6$. No augmented perfect numbers, no augmented sociable numbers were found. Eleven pairs of augmented amicable numbers were found. They are listed in Table 2. Two pairs have both elements over 10^6. They arose from iterating L_+ on 532512, 844740 and 869176.

TABLE 2

L_+

(a)	6160	=	2(4)5·7·11
	11697	=	3·7·557
(b)	12220	=	2(2)5·13·47
	16005	=	3·5·11·97
(c)	23500	=	2(2)5(3)47
	28917	=	3(5)7·17
(d)	68908	=	2(2)7·23·107
	76245	=	3·5·13·17·23
(e)	249424	=	2(4)7·17·131
	339825	=	3·5(2)23·197
(f)	425500	=	2(2)5(3)23·37
	570405	=	3·5·11·3457
(g)	434784	=	2(5)3·7·647
	871585	=	5·11·13·23·53
(h)	649990	=	2·5·11·19·311
	697851	=	3(2)7·11·19·53
(i)	660825	=	3(3)5(2)11·89
	678376	=	2(3)19·4463
(j)	1017856	=	2(11)7·71
	1340865	=	3(2)5·83·359
(k)	1077336	=	2(3)3(2)13·1151
	2067625	=	5(3)7·17·139

REFERENCES

1. R. P. Jerrard and N. Temperley, "Almost Perfect Numbers," *Math. Mag.,* 46 (1973), pp. 84–87.
2. M. Lal and A. Forbes, "A Note on Chowla's Function," *Math. Comp.,* 25 (1971), pp. 923–925. MR 45-6737.

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