Hence, t must be even and $L_r = s^2$. Substituting this result in (5), we obtain: $sL_t - s^2 = 2$, which implies $s|2$, and so $s = 1$ or 2.

SUBCASE A : $s = 1$

Thus, $L_r = 1^2 = 1$, and $r = 1$. Thus, by (2), $F_1 = 1 = F_t$. Since t must be even, thus $t = 2$. Hence, (1,1,2) is another possible solution. Since

$$\frac{1}{\sqrt{5}}\left\{(1+\alpha)^n - (1+\beta)^n\right\} = \frac{1}{\sqrt{5}}\left\{\alpha^{2n} - \beta^{2n}\right\} = F_{2n} = 1^n F_{2n},$$

thus (1,1,2) is a valid solution, the only one yielded by this subcase.

SUBCASE B : $s = 2$

Then $L_r = 2^2 = 4$, so $r = 3$. Thus, by (2), $F_3 = 2 = 2F_t$. As in Subcase A above, $t = 2$. This yields the possible solution (3,2,2). Now

$$(1+\alpha^3) = 2\alpha + 2 = 2\alpha^2;$$

similarly, $(1+\beta^3) = 2\beta^2$. Hence,

$$\frac{1}{\sqrt{5}}\left\{(1+\alpha^3)^n - (1+\beta^3)^n\right\} = \frac{2n}{\sqrt{5}}(\alpha^{2n} - \beta^{2n}) = 2^n F_{2n},$$

which shows that (3,2,2) is indeed a valid solution, the only one yielded by this subcase.

Therefore, all solutions (r,s,t) of the desired identity are given by (7), and also by (1,1,2) and (3,2,2).

Also solved by the Proposer.

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[Continued from page 87.]

Proof. From Corollary 2 and [4, p. 205] we have $s(p^2) = s(p)$ if and only if $f(p^2) = f(p)$ if and only if

$$\phi_{(p-1)/2}(5/9) \equiv 2k(3/2) \pmod{p}.$$

From Wall's remark we note that $\phi_{(p-1)/2}(5/9) \not\equiv 2k(3/2) \pmod{p}$ for all primes p such that $5 < p < 10{,}000$.

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