and

$$0 = \sum_{k=n}^{2n} (-1)^k \sum_{j=0}^{k-n} \binom{n}{j}\binom{n-j}{2n-k} A_{n+j,k} . \tag{5.3}$$

In view of the combinatorial interpretation of $A_{n,k}$ and $G_{n,m}$, (5.2) implies a combinatorial result; however the result in question is too complicated to be of much interest.

For $p = 3$, consider

$$6^n x \frac{G_n^{(3)}(x)}{(1-x)^{3n+1}} = \sum_{k=0}^{\infty} k^n(k^2-1)^n x^k = \sum_{j=0}^{n} (-1)^{n-j}\binom{n}{j} \sum_{k=0}^{\infty} k^{n+2j} x^k = \sum_{j=0}^{n} (-1)^{n-j}\binom{n}{j} \frac{A_{n+2j}(x)}{(1-x)^{n+2j+1}} .$$

Thus we have

$$6^n x G_n^{(3)}(x) = \sum_{j=0}^{n} (-1)^{n-j}\binom{n}{j}(1-x)^{2n-2j} A_{n+2j}(x). \tag{5.4}$$

The right-hand side of (5.4) is equal to

$$\sum_{j=0}^{n} (-1)^{n-j}\binom{n}{j} \sum_{s=0}^{2n-2j} (-1)^s \binom{2n-2j}{s} x^s \sum_{k=1}^{n+2j} A_{n+2j,k} x^k = \sum_{m=1}^{3n} x^m \sum_{j=0}^{n} (-1)^{n-j}\binom{n}{j} \sum_{k=1}^{n+2j} (-1)^{m-k}\binom{2n-2j}{m-k} A_{n+2j,k} .$$

It follows that

$$6^n G_{n,m-1}^{(3)} = \sum_{j=0}^{n} (-1)^{n-j}\binom{n}{j} \sum_{k=1}^{n+2j} (-1)^{m-k}\binom{2n-2k}{m-k} A_{n+2j,k} . \tag{5.5}$$

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Recalling [2, p. 137] that

$$(j+1)\sum_{k=1}^{n} k^j = B_{j+1}(n+1) - B_{j+1} ,$$

where $B_j(x)$ are Bernoulli polynomials with $B_j(0) = B_j$, the Bernoulli numbers, we obtain from (2.3) with $x = 1, B = 1,$ and $C_k = k$ the inequality

$$B_{2p}(n+1) - B_{2p} \leqslant (B_p(n+1) - B_p)^2 \qquad (n = 1, 2, \cdots). \tag{2.4}$$

For $p = 2k+1, k = 1, 2, \cdots, B_{2k+1} = 0,$ and so (2.4) gives the inequality

$$B_{4k+2}(n+1) - B_{4k+2} \leqslant B_{2k+1}^2(n+1) \qquad (n,k = 1, 2, \cdots). \tag{2.5}$$

3. AN INEQUALITY FOR INTEGER SEQUENCES

Noting that $U_k = k$ satisfies the difference equation

$$U_{k+2} = 2U_{k+1} - U_k$$

[Continued on page 151.]