GOLDEN MEAN OF THE HUMAN BODY*

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ABSTRACT

The value of $\phi = (\sqrt{5} + 1)/2$, or 1.61803... is referred to as the Golden Ratio or Divine Proportion. Such a ratio is sometimes discovered in nature, one instance being the mean between lengths of some organs of the human body. Leonardo da Vinci found that the total height of the body and the height from the toes to the navel depression are in Golden Ratio. We have confirmed this by measuring 207 students at the Pascal Gymnasium in Münster, where the almost perfect value of 1.618... was obtained. This value held for both girls and boys of similar ages. However, similar measurements of 252 young men at Calcutta gave a slightly different value—1.615... . The tallest and shortest subjects in the German sample differed in body proportions, but no such difference was noted among the Indians in the Calcutta sample.

INTRODUCTION

Marcus Vitruvius Pollio, Roman architect and author of *De Architecture* (c. 25 B.C.), remarked on a similarity between the human body and a perfect building: "Nature has designed the human body so that its members are duly proportioned to the frame as a whole." He inscribed the human body into a circle and a square, the two figures considered images of perfection. Later (in 1946) Le Corbusier gave a further dimension to the subject by depicting a proportionate human nude (Fig. 1A). In the sketch, he clearly adopted the Fibonacci system and Golden Mean to depict the proportion in a good-looking human body [7]. As shown in the sketch, the figure of a 1.75-meter man with his left hand raised is drawn so that the distance from the foot to the navel measures 108 cm; from the navel to the top of the head measures 66.5 cm; and from the head to the tip of the upraised hand measures 41.5 cm. The ratio between 175 (height of man) and 108 is 1.62, as is the ratio between 108 and 66.5, while the ratio between 66.5 and 41.5 is 1.6. All these means are very close to the Golden Ratio, i.e., $\phi = (\sqrt{5} + 1)/2 = 1.61803...$. In order to verify this fascinating exposition, we set about taking measurements of boys and girls in two remote centers. The experimental subjects showed no visible signs of physical deformity.

MATERIALS AND METHOD

During the last week of October 1973, a group of 207 students (175 boys and 32 girls) at the Pascal Gymnasium in Münster were chosen as subjects for measurement. Also, in early 1974, 252 young men (aged 16-32), most of whom were students at the Indian Statistical Institute in Calcutta, were measured.

The following measurements were taken of bare-footed boys and girls who were asked to stand erect, but without stretching their bodies abnormally,

The authors wish to thank the Director of the Pascal Gymnasium in Münster, W. Germany, for allowing them to record measurements of his pupils and Mr. S. K. De, artist at the Indian Statistical Institute in Calcutta, for making the drawing. Davis is grateful to the German Academic Exchange Service for financial assistance that enabled him to visit Münster in 1973.

against a strong, vertically held pole which was marked in centimeters. With the help of a set-square, three measurements were taken: total height; distance from feet to level of nipples; and distance from feet to navel depression. The following five values were computed from the above three recorded measurements: (A) distance between navel and nipples; (B) distance between nipples and top of head; (C) A + B (navel to top of head); (D) distance from navel to bottom of feet; and (E) total height of subject. Figure 1B illustrates these demarcations. No measurement was made of the distance between the head and the tip of the upraised hand indicated in Le Corbusier's drawing (Fig. 1A).

RESULTS

The German and Indian data were rearranged, separately, in regular descending order, always keeping the tallest subject as first and the shortest subject as last. These data are summarized in Tables 1 and 2.

TABLE 1. BODY MEASUREMENTS OF GERMAN SCHOOL CHILDREN

Particulars	A	B	C	D	E
Total, tallest 50 observations	1127	2136	3263	5335	8618
Total, shortest 50 observations	1010	1757	2767	4354	7121
Grand total (for 207)	4313	8009	12322	19900	32222
Grand Mean	20.836	38.690	59.526	96.135	155.622
Total, girls only	600	1206	1806	2885	4691
Total, boys only	3713	6803	10516	17015	27531

TABLE 2. BODY MEASUREMENTS OF YOUNG MEN FROM CALCUTTA

Particulars	A	B	C	D	E
Total, tallest 63 observations	1496	2729	4225	6678	10903
Total, shortest 63 observations	1314	2348	3662	5885	9547
All men (for 252)	5645	10166	15811	25239	41050
Grand mean	22.40	40.34	62.74	100.15	162.90

Calculated ratios between A & B, B & C, C & D, and D & E are presented in Tables 3 and 4.

TABLE 3: GERMAN STUDENTS: PROPORTION BETWEEN BODY LENGTHS

Population	A/B	B/C	C/D	D/E
Tallest 25% (approximately)	0.528	0.655	0.609	0.621
Shortest 25% (approximately)	0.575	0.635	0.636	0.611
Girls only	0.498	0.668	0.626	0.615
Boys only	0.544	0.647	0.618	0.618
All students (207)	0.537	0.650	0.619	0.618

TABLE 4. CALCUTTA YOUNG MEN: PROPORTION BETWEEN BODY LENGTHS

Population	A/B	B/C	C/D	D/E
Tallest 25%	0.548	0.646	0.633	0.612
Shortest 25%	0.560	0.641	0.622	0.616
All men (252)	0.555	0.643	0.627	0.615

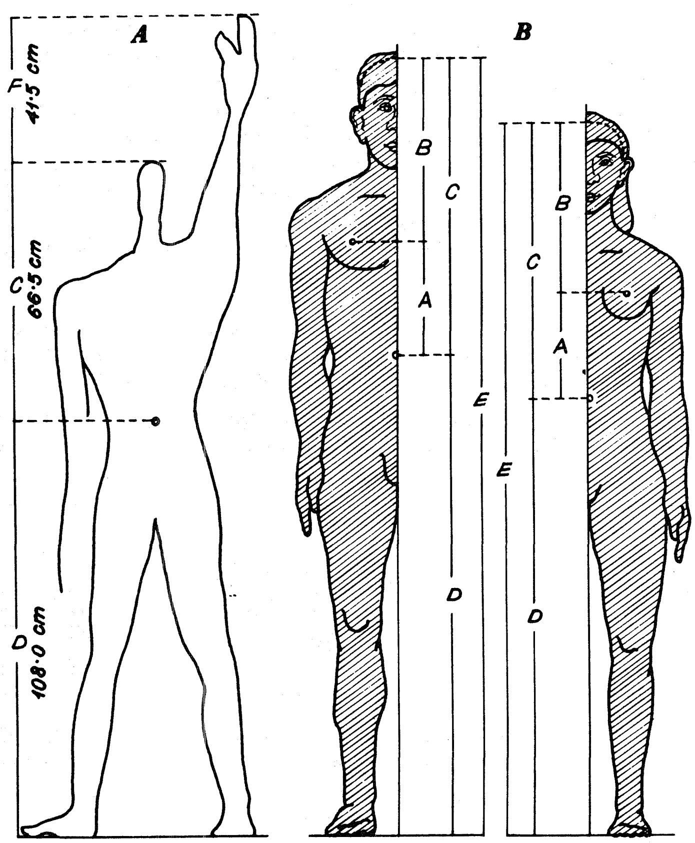

FIGURE 1

Some differences were found to exist between the proportions of corresponding body lengths of the tallest and the shortest subjects. Statistical tests were performed to determine: (1) the extent of the difference; (2) if boys and girls differed in body proportions; and (3) if the Germans differed structurally from the Indians.

STATISTICAL ANALYSIS

For the set of 207 observations on German boys and girls from different age groups, the following statistical hypotheses were tested.

Let U = A/B, V = B/C, W = C/D, X = D/E and let $\overline{u}$, $\overline{v}$, $\overline{w}$, $\overline{x}$ represent the corresponding sample means and the corresponding population means.

There were 27 boys and 32 girls in the same age group in the German sample. Based on their measurements, H_0: $\mu G = \mu B$ was tested. Here

$$\mu G = (\mu uG,\ \mu vG,\ \mu wG,\ \mu xG);\ \mu B = (\mu uB,\ \mu vB,\ \mu wB,\ \mu xB).$$

It is assumed that (U, V, W, X) $N\ (\mu, \Sigma)$. The test statistic used was

$$F = \frac{n_1 + n_2 - 5}{4} \cdot \frac{1}{(1/n_1 + 1/n_2)} \quad (\underset{\sim}{\overline{Y}}_G - \underset{\sim}{\overline{Y}}_B)'A^{-1}(\underset{\sim}{\overline{Y}}_G - \underset{\sim}{\overline{Y}}_B),$$

which is distributed as an F statistic with 4, $n_1 + n_2 - 5$, d.f.

$$\overline{Y}_G = (\overline{U}_G, \overline{V}_G, \overline{W}_G, \overline{X}_G)\ ;\ \overline{Y}_B = (\overline{U}_B, \overline{V}_B, \overline{W}_B, \overline{X}_B)\ .$$

$A = A_1 + A_2$, A_i = sum of squares and products matrix for the ith population, i = 1, 2.

$$\overline{Y}_G = (.5962,\ .6300,\ .6303,\ .6137)$$
$$\overline{Y}_B = (.4883,\ .6460,\ .7883,\ .5769)$$
$$F = 4.49,\ F_{.05};\ 4.50 = 5.70.$$

So, H_0 is accepted at the 5% level of significance, i.e., there is no significant difference between measurements of girls and boys. However, the test for H_0: $\mu u' = \mu u^2$ gave an insignificant value for the t statistic, which was less than 1.

Again, H_0: $\mu = \mu^2$ was tested for the 50 tallest and the 50 shortest individuals, where $\mu i = (\mu ui,\ \mu vi,\ \mu wi,\ \mu xi)$.

$$\overline{\underset{\sim}{Y}}_1 = (.5196,\ .6417,\ .5979,\ .6089) \text{ for 50 tallest;}$$

$$\overline{\underset{\sim}{Y}}_2 = (.5817,\ .6350,\ .6368,\ .6114) \text{ for 50 shortest.}$$

The computed $F = 10.1574$ and $F_{.05}$; $4.95 = 5.66$, $F_{.01}$; $4.95 = 14.57$, so H_0 is rejected at the 5% level of significance.

Next, H_0: $\mu u' = \mu u^2$ was rejected at both the 5% and 1% levels of significance because $t = -2.93$ with 98 d.f. Also, H_0: $\mu w' = \mu w^2$ was rejected (at both levels) because $t = -3.3$ with 98 d.f.

For the Indian data, H_0: $u' = u^2$ was tested for the 63 tallest and the 63 shortest subjects (25% of the total). For this both H_0: $\mu u' = \mu u^2$ and H_0: $\mu w' = \mu w^2$ were accepted because the corresponding t statistics were < 1.

Again for the Indian data we did not find any significant difference between measurements of the tallest and shortest subjects. This might have been due to the short range of heights among the Indian sample.

Indian Data	German Data
$\overline{E}_1 = 173.38,\ \overline{E}_2 = 152.95$	$\overline{E}_1 = 172.36,\ \overline{E}_2 = 142.42$

H_0: $\mu E_1 = \mu E_2$ was rejected because $t \simeq 5$ with 111 d.f.; i.e., the heights of the shortest individuals in the Indian sample and those in the German sample differed significantly. The variance in mean ages of the two sample groups might also be an important reason for the difference.

DISCUSSION

The data on German students presented in Tables 1 and 3 confirm La Corbusier's definition of a good-looking human body.

The Parthenon at Athens is considered one of the most perfect buildings ever constructed by man and one that has survived centuries of neglect. The secret lies in the fact that the Parthenon was constructed according to the principle of Divine Proportion [4]. The width of the building and its height are in Golden Sections. Hoggatt [3] has cited further examples in which the Golden Section has been used.

Also, it is now known [see 5] that the Great Pyramid of Giza, Egypt, was built in accordance with Divine Proportion; its vertical height and the width of any of its sides are in Golden Sections.

These examples confirm Vitruvius' statement that perfect buildings and proportionate human bodies have something in common.

According to available data, the navel of the human body is a key point that divides the entire length of the body into Golden Sections (their ratio is the Golden Ratio). This point is also vitally important for the developing fetus, since the umbilical cord—the life-line between mother and fetus—is connected through the navel. Compared to the position of the navel, the line of the nipples is not particularly important, because it does not divide the body (above the navel) into Golden Sections. Data from both Germany and India confirm this fact.

There is a close connection between the Golden Ratio and the Fibonacci Sequence—1, 1, 2, 3, 5, 8, 13, 21, ... Each number is obtained by adding the two numbers just previous to it. This numerical sequence is named after the thirteenth-century Italian mathematician Leonardo Pisano, who discovered it while solving a problem on the breeding of rabbits. Ratios of successive pairs of some initial numbers give the following values:

1/1 = 1.000; 1/2 = 0.500; 2/3 = 0.666...; 3/5 = 0.600; 5/8 = 0.625;
8/13 = 0.615...; 13/21 = 0.619...; 21/34 = 0.617...; 34/55 = 0.618...;
55/89 = 0.618... .

Thereafter, the ratio reaches a constant that is almost equivalent to the Golden Ratio. Such a ratio has been detected in most plants with alternate (spiral) phyllotaxis, because any two consecutive leaves subtend a Fibonacci angle approximating 317.5 degrees. Thus, many investigators of phyllotaxis identify the involvement of Fibonacci series on foliar arrangement, the most recent being Mitchison [6].

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A RECURRENCE RELATION FOR GENERALIZED MULTINOMIAL COEFFICIENTS

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1. INTRODUCTION

Gould [2] has defined Fontené-Ward multinomial coefficients by

$$\left\{ {n \atop s_1,\ s_2,\ \ldots,\ s_r} \right\} = u_n!/u_{s_1}!u_{s_2}!\ \ldots\ u_{s_r}!$$

where $\{u_n\}$ is an arbitrary sequence of real or complex numbers such that

$$u_n \neq 0 \text{ for } n \geq 1,$$

$$u_0 = 0,$$

$$u_1 = 1,$$

and $\qquad u_n! = u_n u_{n-1} \ldots u_1,$

with $\qquad u_0! = 1.$

These are a generalization of ordinary multinomial coefficients for which there is a recurrence relation

$$\left({n \atop s_1,\ \ldots,\ s_r} \right) = \sum_{j=1}^{r} \left({n-1 \atop s_1 - \delta_{1j},\ \ldots,\ s_r - \delta_{rj}} \right)$$

as in Hoggatt and Alexanderson [4].

Hoggatt [3] has also studied Fontené-Ward coefficients when $r = 2$ and $\{u_n\} = \{F_n\}$, the sequence of Fibonacci numbers. We propose to consider the case where the u_n are elements which satisfy a linear homogeneous recurrence relation of order r.