A NOTE ON THE MULTIPLICATION OF TWO 3 X 3 FIBONACCI-ROWED MATRICES

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A Fibonacci-rowed matrix is defined to be a matrix in which each row consists of consecutive Fibonacci numbers in increasing order.

Laderman [1] presented a noncommutative algorithm for multiplying two 3 x 3 matrices using 23 multiplications. It still needs 18 multiplications if Laderman's algorithm is applied to the product of two 3 x 3 Fibonacci-rowed matrices. In this short note, an algorithm is developed in which only 17 multiplications are needed. This algorithm is mainly based on Strassen's result [2] and the fact that the third column of a Fibonacci-rowed matrix is equal to the sum of the other two columns.

Let $C = AB$ be the matrix of the multiplication of two 3 x 3 Fibonacci-rowed matrices. Define

$$\begin{aligned}
\mathrm{I} &= (a_{11} + a_{22})(b_{11} + b_{22})\\
\mathrm{II} &= a_{23}b_{11}\\
\mathrm{III} &= a_{11}(b_{12} - b_{22})\\
\mathrm{IV} &= a_{22}(-b_{11} + b_{21})\\
\mathrm{V} &= a_{13}b_{22}\\
\mathrm{VI} &= (-a_{11} + a_{21})b_{13}\\
\mathrm{VII} &= (a_{12} - a_{22})b_{23}
\end{aligned}$$

Then

$$C = \begin{bmatrix} \mathrm{I} + \mathrm{IV} - \mathrm{V} + \mathrm{VII} + a_{13}b_{31} & \mathrm{III} + \mathrm{V} + a_{13}b_{32} & c_{11} + c_{12} \\ \mathrm{II} + \mathrm{IV} + a_{23}b_{31} & \mathrm{I} + \mathrm{III} - \mathrm{II} + \mathrm{VI} + a_{23}b_{32} & c_{21} + c_{22} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & c_{31} + c_{32} \end{bmatrix}.$$

There are only 17 multiplications involved in calculating . However, 18 multiplications are needed if Laderman's algorithm [1] is applied, namely

$$m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_{11}, m_{12},$$
$$m_{13}, m_{14}, m_{15}, m_{16}, m_{17}, m_{19}, m_{20}, m_{22}$$

(see [1]). In fact, only 18 multiplications are needed if the usual process of multiplication is applied.

REFERENCES

1. Julian D. Laderman. "A Noncommutative Algorithm for Multiplying 3 × 3 Matrices Using 23 Multiplications." *Bull. A.M.S.* 82 (1976):126-128.
2. V. Strassen. "Gaussian Elimination Is Not Optimal." *Numerische Math.* 13 (1969):354-356.

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