

Given triangle ABC with $AB = \alpha$, $BC = \beta$, $CA = \gamma$, and circles with centres A, B, and C having radii a , b , and c , respectively.

Let $\ell = a + b + \alpha$; $m = b + c + \beta$; $n = \alpha + b - a$; $p = \beta + b - c$;
 $q = a + b - \alpha$; $t = b + c - \beta$; $u = \alpha + a - b$; $v = \beta + c - b$;
 $s = (\alpha + \beta + \gamma)/2$.

Then, if x is the radius of a circle touching the three given ones:

$$4(x + b)\sqrt{s(s - \gamma)} = \sqrt{np(2x + \ell)(2x + m)} \pm \sqrt{uv(2x + q)(2x + t)}$$

the positive sign being taken if the centre of the required circle falls outside angle ABC, and the negative sign if it falls inside angle ABC.

The formula applies to *external* contact. If a given circle of radius a , say, is to make *internal* contact with the required one, then $-a$ must replace $+a$ in the formula. If a given circle of radius a , say, becomes a point, put $a = 0$.

When the three given circles touch each other externally,

$$\alpha = a + b, \beta = b + c, \text{ and } \gamma = a + c,$$

and the above formula yields the solution mentioned by Trigg, viz.

$$x = abc/[2\sqrt{abc(a + b + c)} \pm (ab + bc + ca)].$$

LETTER TO THE EDITOR

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Dear Professor Hoggatt,

In a recent article with Claudia Smith [*Fibonacci Quarterly* 14 (1976): 343], you referred to the question whether a prime p and its square p^2 can have the same rank of apparition in the Fibonacci sequence, and mentioned that Wall (1960) had tested primes up to 10,000 and not found any with this property.

I have recently extended this search and found that no prime up to one million (1,000,000) has this property.

My computations in fact test the Lucas sequence for the property

$$(1) \quad L_p \equiv 1 \pmod{p^2} \quad p = \text{prime.}$$

For $p > 5$, this is easily shown to be a necessary and sufficient condition for p and p^2 to have the same rank of apparition in the Fibonacci sequence, because of the identity

$$(2) \quad (L_p - 1)(L_p + 1) = 5F_{p-1}F_{p+1}.$$

So far, I have shown that the congruence (1) does not hold for any prime less than one million; I hope to extend the search further at a later date.

You may wish to publish these results in *The Fibonacci Quarterly*.

Yours sincerely,

[Dr L. A. G. Dresel]
