using the properties of the U-nary representation and Lemma 1 of the solution to B-421. This contradiction establishes the only remaining possibility, i.e., $c_k = 0$, $d_k = 1$. This establishes the desired result.

Also solved by Sahib Singh and the proposer.

Telescoping Infinite Product

B-423 *Proposed by Jeffery Shallit, Palo Alto, CA*

Here let F_n be denoted by $F(n)$. Evaluate the infinite product

$$\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{13}\right)\left(1 + \frac{1}{610}\right) \cdots = \prod_{n=1}^{\infty}\left[1 + \frac{1}{F(2^{n+1} - 1)}\right].$$

Solution by Gregory Wulczyn, Bucknell University, Lewisburg, PA

Let L_n also be written as $L(n)$ and $A_n = 1 + [1/F(2^{n+1} - 1)]$. It is easily seen (for example, from the Binet formulas) that

$$L(2)L(4)L(8) \ldots L(2^n) = F(2^{n+1}) \quad \text{and} \quad 1 + F(2^{n+1} - 1) = F(2^n - 1)L(2^n).$$

Hence, $A_n = F(2^n - 1)L(2^n)/F(2^{n+1} - 1)$ and

$$\prod_{i=1}^{\infty} A_n = \lim_{n\to\infty} \frac{F(1)F(3)F(7)F(15) \cdots F(2^n - 1)L(2)L(4)L(8) \cdots L(2^n)}{F(3)F(7)F(15) \cdots F(2^{n+1} - 1)}$$

$$= \lim_{n\to\infty} \frac{F(2^{n+1})}{F(2^{n+1} - 1)},$$

and the desired limit is $\alpha = (1 + \sqrt{5})/2$.

Also solved by Paul S. Bruckman, Bob Prielipp, and the proposer.

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(Continued from page 6)

Hence

$$u_{n-1} = x_1 u_n - Dy_1 v_n = (x_1 - 1)u_n - Dy_1 v_n + u_n \geq u_n.$$

Thus $n = 0$.

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