holds for all polynomials p. With $p(u) = u^{n+1-d}$ it follows from (6) and (7) that

$$C_n^{(d)} = L\big((u)_{d-1}(u - d + 1)^{n+1-d}\big) = L(u^{n+1-d}) = B_{n+1-d}.$$

It is possible to construct a bijection φ from the partitions of $\overline{n}$ to the d-Fibonacci partitions of $\overline{n + d - 1}$ in a way similar to that given in the previous section; however, this is more complicated to describe and therefore is omitted.

3. A GENERALIZATION OF THE FIBONACCI NUMBERS

The fact that F_{n+1} is the number of Fibonacci subsets of $\overline{n}$ can be seen as the starting point to define the numbers $F_n^{(s)}$ $(s \in N)$:

$F_{n+1}^{(s)}$ is defined to be the number of $(A_1, \ldots, A_s)$ with $A_i \subseteq \overline{n}$ and $A_i \cap A_j \neq \emptyset$ for $i \neq j$. The recurrence

$$F_{n+1}^{(s)} = sF_n^{(s)} + F_{n-1}^{(s)},\ F_1^{(s)} = 1,\ F_2^{(s)} = 1 + s$$

can be established as follows:

First, $F_{n+1}^{(s)}$ can be expressed as the number of functions

$$f : \overline{n} \to \{\varepsilon, a_1, \ldots, a_s\}$$

with $f(i) = f(i + 1) = a_j$ is impossible. If $f(n) = \varepsilon$, the contribution to $F_{n+1}^{(s)}$ is $F_n^{(s)}$. If $f(n) = a_i$, the contribution is $F_n^{(s)}$ minus the number of functions

$$f\colon \overline{n - 1} \to \{\varepsilon, a\ , \ldots, a_s\}$$

with $f(n - 1) = a_i$. Taken all together,

$$F_{n+1}^{(s)} = F_n^{(s)} + s[F_n^{(s)} - F_{n-1}^{(s)} + F_{n-2}^{(s)} - + \cdots]. \tag{8}$$

Also

$$F_{n+2}^{(s)} = F_{n+1}^{(s)} + s[F_{n+1}^{(s)} - F_n^{(s)} + F_{n-1}^{(s)} - + \cdots]. \tag{9}$$

Adding (8) and (9) gives the result. An explicit expression is

$$F_n^{(s)} = \frac{1}{\sqrt{s^2 + 4}}\left[\left(\frac{s + \sqrt{s^2 + 4}}{2}\right)^{n+1} - \left(\frac{s - \sqrt{s^2 + 4}}{2}\right)^{n+1}\right].$$

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(continued from page 406)

Added in proof. Other explicit formulas for $P(n, s)$ were obtained in the paper "Enumeration of Permutations by Sequences," *The Fibonacci Quarterly* 16 (1978): 259-68. See also L. Comtet, *Advanced Combinatorics* (Dordrecht & Boston: Reidel, 1974), pp. 260-61.

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