EXPLORING FIBONACCI RESIDUES

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Mathematicians have developed a simple and powerful method of relating numbers to each other from the standpoint of division. They say that two numbers a and b are congruent to each other modulo m when the difference a - b is divisible by m. It can be readily shown that this is equivalent to the statement that on dividing a and b by m, they will both give the same "remainder" — which the mathematician calls the least positive residue.

What remainder is obtained when we divided one Fibonacci number by another? The remainder could of course be zero, but as is well known, zero is one of the Fibonacci numbers. Do we always obtain a Fibonacci number for the least positive residue? If not, will we obtain a Fibonacci number if we allow the use of either the least positive or the least negative residue?

This is the general line of investigation. In cases in which Fibonacci numbers are the result, the investigator should seek to find some type of regularity and thus formulate a mathematical theorem. Once this has been done a proof is a desideratum.

Just to start the process let us find a few least positive and least negative residues. Using $F_{20}(6765)$ as the dividend and various Fibonacci numbers as divisors, we find that

$$\begin{array}{lll} 6765 \equiv 33 \pmod{34} & 6765 \equiv 8 \pmod{233} & 6765 \equiv 843 \pmod{987} \\ 6765 \equiv 0 \pmod{55} & 6765 \equiv 356 \pmod{377} & 6765 \equiv 377 \pmod{1597} \\ 6765 \equiv 141 \pmod{144} & 6765 \equiv 55 \pmod{610} & \end{array}$$

It seems that in some cases the least positive residue is a Fibonacci number whereas in others apparently it is not. In the latter, we go to the least negative residue, we apparently get Fibonacci numbers in these cases as well. Thus $6765 \equiv -21 \pmod{377}$; $6765 \equiv -144 \pmod{987}$.

Those making discoveries in regard to this problem are encouraged to send their findings to Brother U. Alfred, St. Mary's College, Calif., by July 31st so that they may be published in the October, 1964, issue of the Quarterly.