FIBONACCI NUMBERS: THEIR HISTORY THROUGH 1900

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In 1202, a remarkable man wrote a remarkable book. The man was Leonardo of Pisa, known as Fibonacci, a brilliant man in an intellectual wilderness. The book Liber Abacci (The Book of the Abacus) introduced Arabic numbers into Europe.

In the book was a seemingly simple little problem:

"A pair of rabbits are enclosed on all sides by a wall. To find out how many pairs of rabbits will be born in the course of one year, it being assumed that every month a pair of rabbits will produce another pair, and that rabbits begin to bear young two months after their own birth."

On the margin of the manuscript, Fibonacci gives the tabulation:

A pair
1
First
2
Second
3
Third
5
Fourth
8
Fifth
13
Sixth
21
Seventh
34
Eighth
55
Ninth
89
Tenth
144
Eleventh
233
Twelfth
377

He sums up his calculations

"... we see how we arrive at it. We add to the first number the second one, i. e., 1 and 2; the second to the third; the third to the fourth; the fourth to the fifth; and in this way, one after another, until we add together the tenth and eleventh and obtain the total number of rabbits — 377; and it is possible to do this in this order for an infinite number of months."

There the matter lay for 400 years. In 1611, Johann Kepler [1] of astronomy fame, arrived at the series 1, 1, 2, 3, 5, 8, 13, 21, ... There is no indication that he had access to one of Fibonacci's hand-written books (The Liber Abacci was not published until 1857 [2]). At any rate, in discussing the Golden Section and phyllotaxis, Kepler wrote:

"For we will always have as 5 is to 8 so is 8 to 13, practically, and as 8 is to 13, so is 13 to 21 almost. I think that the seminal faculty is developed in a way analogous to this proportion which perpetuates itself, and so in the flower is displayed a pentagonal standard, so to speak. I let pass all other considerations which might be adduced by the most delightful study to establish this truth."

Simon Stevens (1548-1620) also wrote on the Golden Section. The editor of his works, A. Gerard [3] arrived at the formula for expressing the series in 1634

$$U_{n+2} = U_{n+1} + U_n$$

A hundred years must pass before the problem is again considered. In 1753, R. Simpson [4] derived a formula, implied by Kepler

$$U_{n-1} U_{n+1} - U_n^2 = (-1)^{n+1}$$

A second hundred years pass by and the series again comes under study. In 1843, J. P. M. Binet [5] derives an analytical function for determining the value of any Fibonacci number

$$2^n \sqrt{5}\, U_n = (1 + \sqrt{5})^n - (1 - \sqrt{5})^n$$

The following year, B. Lamé [6] first used the series to solve a problem in Theory of Numbers. He investigated the number of operations needed to find the GCD of two integers (it does not exceed 5 times the number of digits in the smaller number).

Two years later, E. Catalan [7] derived the important formula

$$2^{n-1}U_n = \frac{n}{1} + \frac{5n(n-1)(n-2)}{1\cdot 2\cdot 3} + \frac{5^2 n(n-1)(n-2)(n-3)(n-4)}{1\cdot 2\cdot 3\cdot 4\cdot 5} + \ldots$$

By now, the series had received enough attention to deserve a name. It was variously called the Braun Series, the Schimper-Braun series, the Lamé series and the Gerhardt series.

A. Braun [8], applied the series to the arrangement of the scales of pine cones. Schimper is completely unknown. Lamé has already been mentioned, but the name has been credited to Father Bernard Lami, a contemporary of Newton and the discoverer of the parallelogram of forces. Gerhardt is probably a mis-spelling of Girard.

Edouard Lucas [20], who dominated the field of recursive series during the period 1876-1891, first applied Fibonacci's name to the series and it has been known as Fibonacci series since then.

About this time, 1858, Sam Loyd claimed to have invented the checkerboard paradox [9]. It is first found in print in a German journal in 1868 [10]. Today it seems proper to call it the Carroll Paradox after Lewis Carroll [11] (Charles Dodgson, 1832-93) who was quite fond of it.

Before the century ended, a number of familiar relations were found. Among them: (V_n is the nth Lucas number.)

1876, E. Lucas [12]

$$U_{n-1}^2 + U_n^2 = U_{2n+1}$$

$$V_{4n} = U_{2n}^2 - 2$$

$$V_{4n+2} = U^2_{2n+1} + 2$$

$$U^{n+p} = U^{n-p} (U+1)^p$$

$$U^{n-p} = U^n (U-1)^p$$

1886, E. Catalan [13] [14]

$$U_{n+1-p} U_{n+1+p} - U^2_{n+1} = (-1)^{n+2-p} U_p$$

$$U^2_n - U_{n-p} U_{n+p} = (-1)^{n-p+1} U_{n-1}$$

1899, E. Landau [15] related the series

$$\sum_{n=1}^{\infty} (1/U_{2n})$$

to Lambert's series and

$$\sum_{n=0}^{\infty} (1/U_{2n+1})$$

to the theta series.

A complete list can be found in Vol. 1 of Dickson's "History of the Theory of Numbers."

This small history ends arbitrarly at 1900 for the pregmatic reason that a mere listing of twentieth century developments would fill a moderately sized volume. It would be interesting to see Fibonacci's reaction to the application of his rabbit problem to such diverse subjects as musical composition [16], process optimization [17], electrical network theory [18], and genetics [19].

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