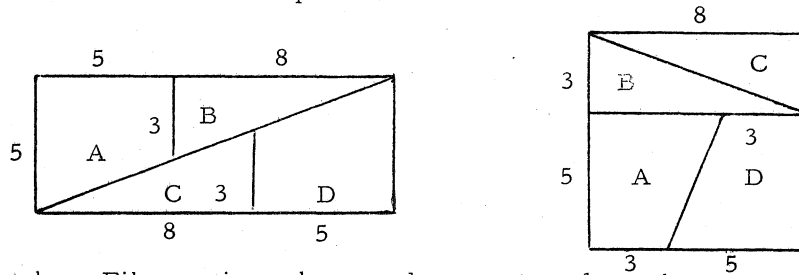


THE VANISHING SQUARE

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This well-known puzzle:



involves Fibonacci numbers and suggests a formula:

$$|F_{i-1}F_{i+1} - F_iF_i| = F_1$$

(Absolute value, since $3 \times 8 - 5 \times 5 = -1$ but $5 \times 13 - 8 \times 8 = +1$) Adding and subtracting $F_{i-1}F_i$:

$$\begin{aligned} &F_{i-1}F_{i+1} - F_{i-1}F_i + F_{i-1}F_i - F_iF_i \\ &F_{i-1}(F_{i+1} - F_i) + F_i(F_{i-1} - F_i) \\ &F_{i-1}F_{i-1} - F_iF_{i-2} \end{aligned}$$

Repeating the process with $F_{i-2}F_{i-1}$:

$$\begin{aligned} &F_{i-1}F_{i-1} - F_{i-2}F_{i-1} + F_{i-2}F_{i-1} - F_iF_{i-2} \\ &F_{i-1}(F_{i-1} - F_{i-2}) + F_{i-2}(F_{i-1} - F_i) \\ &F_{i-1}F_{i-3} - F_{i-2}F_{i-2} \end{aligned}$$

After a finite number of steps, the smallest subscript becomes 1, and:

$$F_3F_1 - F_2F_2 \quad \text{or} \quad F_2F_2 - F_3F_1 \quad \text{or} \quad \pm F_1$$

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