Mixed Convolution

B-507 *Proposed by Heinz-Jürgen Sieffert, Berlin, Germany*

Let G_n and H_n be as in B-506. Find a formula for $\sum_{k=0}^{n} G_k H_{n-k}$ similar to the formulas in B-506.

Solution by Paul S. Bruckman, Fair Oaks, CA

We follow the notation introduced in the solution to B-506, and note that

$$A(x)B(x) = \sum_{n=0}^{\infty} G_n x^n \cdot \sum_{n=0}^{\infty} H_n x^n = \sum_{n=0}^{\infty} x^n \sum_{k=0}^{n} G_k H_{n-k}.$$

On the other hand,

$$A(x)B(x) = 5^{-1/2}(P^4 - Q^4) = 5^{-1/2} \sum_{n=0}^{\infty} \binom{n+3}{3} (\alpha^n - \beta^n) x^n$$

$$= \frac{1}{6} \sum_{n=0}^{\infty} (n+2)(n+3) G_n x^n.$$

Hence,

$$\sum_{k=0}^{n} G_k H_{n-k} = \frac{1}{6}(n+2)(n+3)G_n.$$

Also solved by C. Georghiou, L. Kuipers, J. Suck, Gregory Wulczyn, and the proposer.

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(Continued from page 272)

3. A. F. Horadam. "Geometry of a Generalized Simson's Formula." *The Fibonacci Quarterly* 20, no. 2 (1982):164-68.
4. A. G. Shannon & A. F. Horadam. "Infinite Classes of Sequence-Generated Circles." *The Fibonacci Quarterly* (to appear).
5. L. G. Wilson. "Fibonacci Sequences." Private communication, 1982.

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