THE GOOSE THAT LAID THE GOLDEN EGG

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Last year when I was 13 and we were studying elementary algebra, I learned that $x + y = 1$ could be graphed as a line with x- and y-intercepts of (1, 0) and (0, 1) and that $x^2 + y^2 = 1$ could be graphed as a circle of radius 1 with its center at (0, 0). This year we studied functions of the form

$$f(x) = Ax^2 + Bx + C$$

and saw that their graphs were parabolas. Since this shape was so different from a circle, with what did not appear to me to be an enormous difference in mathematical form, I wondered what other curves of the form $x^n + y^n = 1$ would look like. Fortunately, I have an Atari 800 computer at home which allows me the opportunity to make such an investigation relatively simple.

Eventually, I became bored with integral exponents and, since I had been working with the golden ratio for a math project, I wondered what $x^{\phi^2} + y^{\phi^2} = 1$, where ϕ = the Golden Ratio, 1.618033989..., would look like. Inasmuch as all other facets of this ratio that I had investigated were so special, I thought that graphs using it should have very interesting shapes. I was correct. As soon as I looked at the shape generated by $x^{\phi^2} + y^{\phi^2} = 1$, I recognized it as one end of an egg. This was an amazement to me. Knowing that eggs do not have two axes of symmetry, I wondered whether I could combine the curve generated with the curve of a slightly altered function to create the rest of a realistically-shaped egg. My hypothesis was that there is an egg shape (which I called the "golden egg") whose configuration is directly related to the golden ratio. It is a composite shape, different from a circle, ellipse, or oblate spheroid. The left portion of the egg is the graph of the function generated by the golden ratio exponential $x^{\phi^2} + y^{\phi^2} = 1$. The right portion of the egg is the graph of the function generated by the golden ratio exponential:

$$x^{\phi} + \frac{1}{\phi^{\phi}}(y)^{\phi} = 1 \quad \text{(see Figure 1).}$$

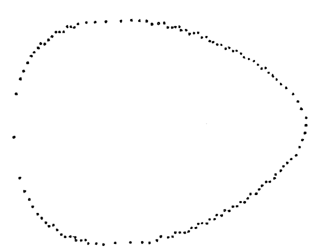
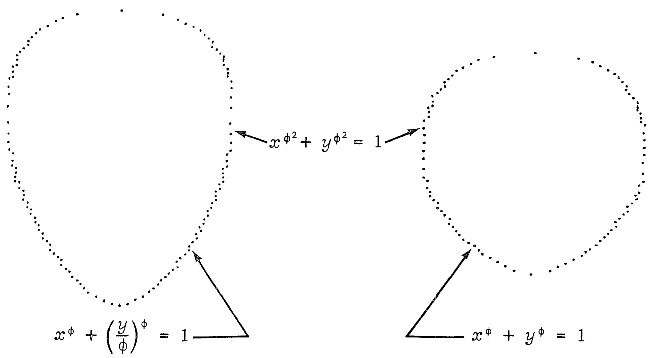


FIGURE 1. Picture of Golden Egg Generated by Atari 800 Computer

I was so pleased with the result of my graph and my development and analysis of the golden egg that I wondered whether Fibonacci-related exponential functions would generate other configurations. I began to experiment with the coefficients and the positions of x and y. By rotating the golden egg, I noted that the curve showed a strong resemblance to the shape of an adult head. A change in the y-coefficient created the outline of an infant's head. (See Figure 2.)



Adult Shape Infant Shape

FIGURE 2

With additional changes to the coefficients and constants, carrots, acorns, pine cones, and other figures appeared on my computer console. These figures, with descriptions of the equations used appear below as Figures 3 through 7.

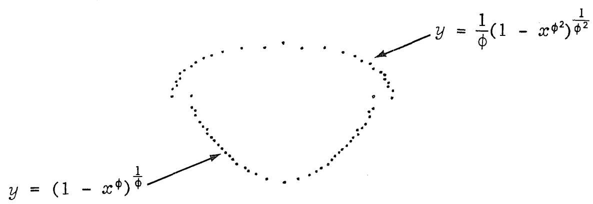


FIGURE 3. Acorn

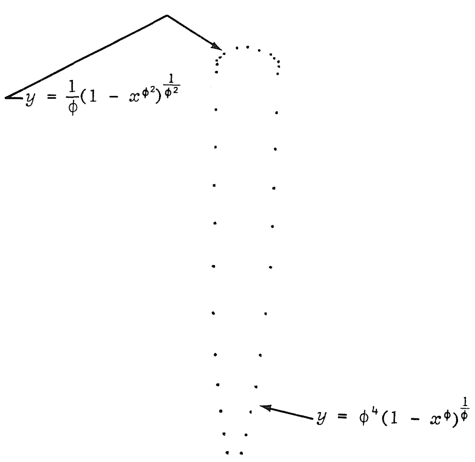


FIGURE 4. Carrot

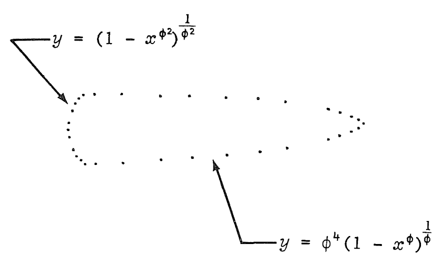
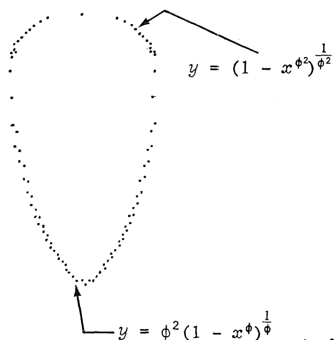
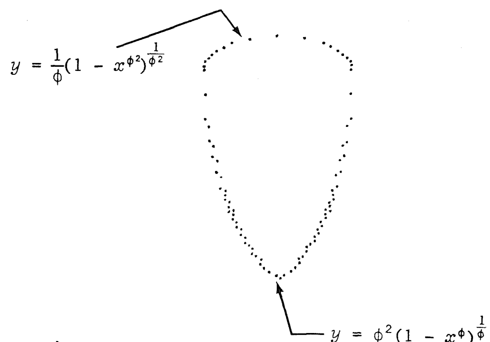


FIGURE 5. Corn on the Cob





(ϕ'*N* Cones)

FIGURE 6. Round-Top Pine Cone

FIGURE 7. Flat-Top Pine Cone

It is interesting that Brother Alfred Brousseau found Fibonacci numbers in pine cones, and now we find pine cones in Fibonacci-related functions [1].

I have defined equations of the type used to generate the previous configurations as Golden Functions (i.e., equations that are functions of variables raised to a power that is a function of ϕ). One might wonder whether the creation of the Golden Functions and these shapes is merely an academic exercise and an accident. I choose to believe not and leave the investigation of equations of the form $Ax^{\phi} + Bx + C = 1$ and $x^{\phi}y = 1$ to the reader.

REFERENCE

1. Brother Alfred Brousseu. *Fibonacci Numbers in Nature*. Santa Clara, Calif.: The Fibonacci Association, 1965, p. 7.

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