AN EASY PROOF OF THE GREENWOOD-GLEASON EVALUATION OF THE RAMSEY NUMBER $R(3, 3, 3)$

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1. INTRODUCTION

In 1955, Greenwood & Gleason proved that the Ramsey number $R(3, 3, 3) = 17$ by constructing a triangle-free, edge-chromatic graph in three colors of order 16. Their method employed finite fields. This result was obtained later by another method. Here, we give yet another method which can be called "group-theoretical" or, merely, "adding binary codes."

2. THE METHOD

Consider the set of 16 binary codes {0000, 0001, 0010, 0011, ..., 1111}; if we add them componentwise with $0 + 0 = 0$, $1 + 0 = 0 + 1 = 1$, and $1 + 1 = 0$, then this set G under + is isomorphic to the elementary abelian group of order 16. Partition the 15 nonidentity elements into three sets G_1, G_2, G_3 so that no two elements in any of the three sets add up to an element in the same set. Then, we identify the vertices of a graph Γ with the elements of this group G. We 3-color the edges as follows: join the vertices x and y by an edge of color i if $x + y \in G_i$; join 0000 with x by an edge of color i if $x \in G_i$.

3. THE CONSTRUCTION

Partition the 15 nonidentity elements into 3 sets:

$$G_1 = \{1100, 0011, 1001, 1110, 1000\},$$

$$G_2 = \{1010, 0101, 0110, 1101, 0100\},$$

$$G_3 = \{0001, 0010, 0111, 1011, 1111\}.$$

Obviously, no two elements in G_i add up to be an element in G_i. We thus obtain:

4. THE GRAPH

Using solid lines for color 1, dot-dash lines for color 2, and dotted lines for color 3, the triangle-free, edge-chromatic graph in three colors of order 16 is shown in Figures 1-4.

5. EXTENSION OF THE METHOD

This method *can* be used to find the lower bound of other Ramsey numbers. to this end, one first finds an appropriate group, partitions the group elements into several subsets, making sure that in each subset the product of two elements is never in it. The sharpness of the bound depends on the choice of the group.

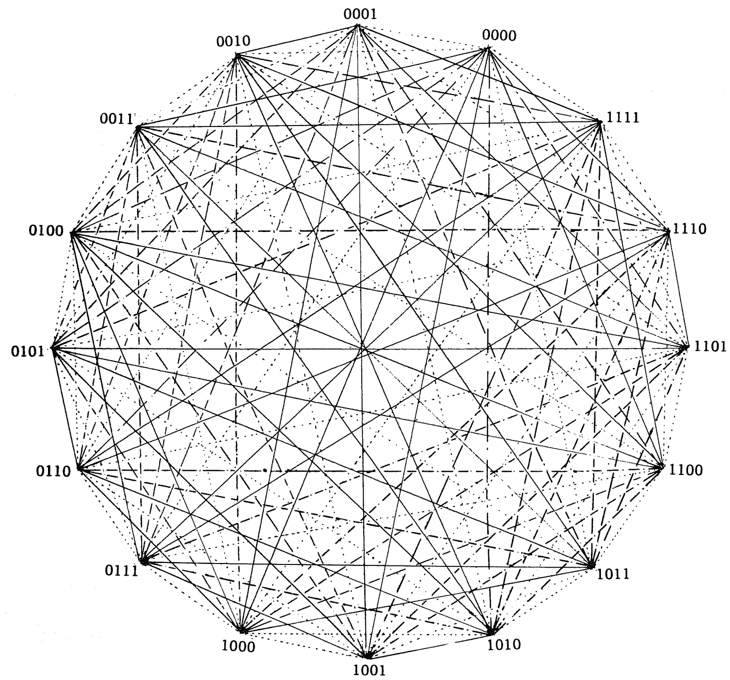

FIG. 1. The Triangle-Free, Edge-Chromatic Graph in Three Colors of Order 16

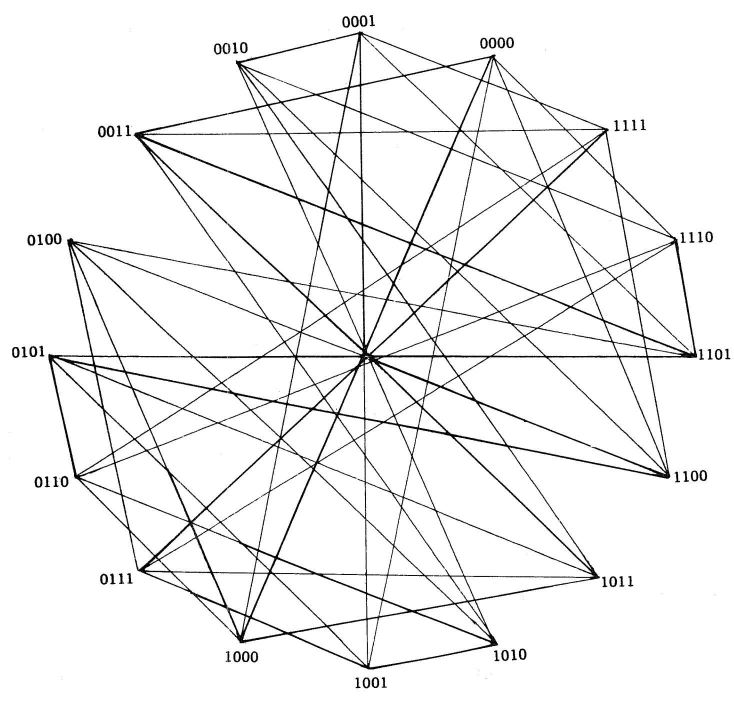

FIG. 2. Subgraph of Solid Lines

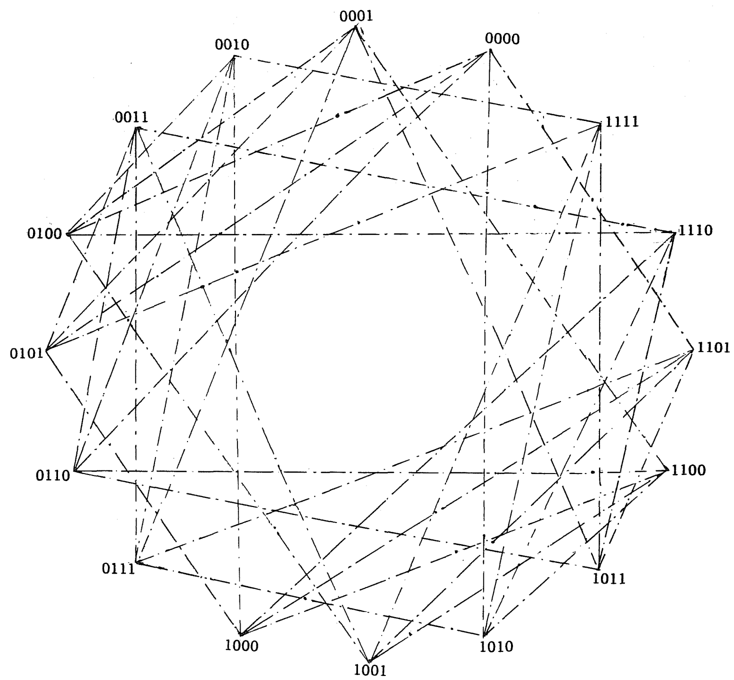

FIG. 3. Subgraph of Dot-Dash Lines

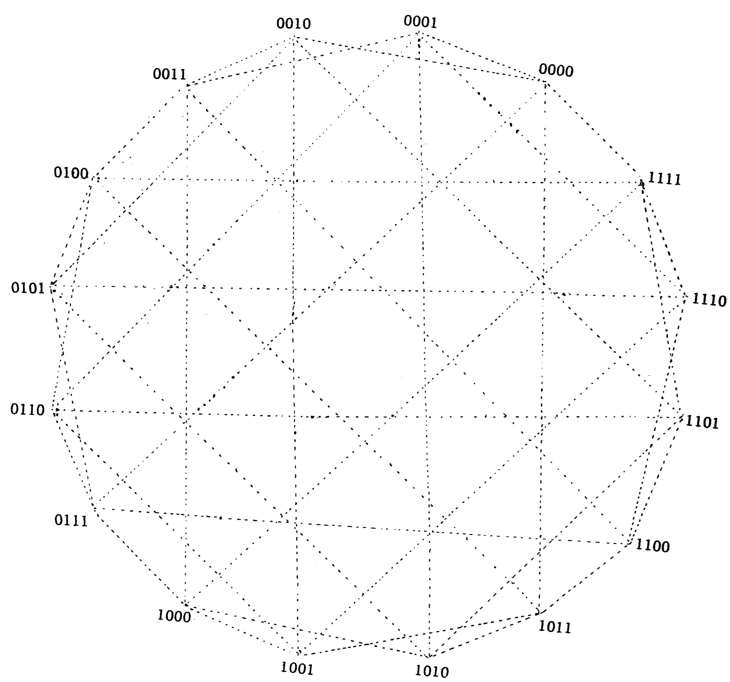

FIG. 4. Subgraph of Dotted lines

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