A. F. HORADAM—AD MULTOS ANNOS

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1. INTRODUCTION

"The sequence will involve a cumulative process of capital appreciation that will accrue to the benefit of the early rich and their heirs" [2]. Though very much out of context, the quotation came to mind when I sat down to respond to the Editor's invitation to write a paper about Alwyn Horadam, an elder states-man of The Fibonacci Association, on his retirement. As a former student, or intellectual heir, of "Horrie" (as he was affectionately known to thousands of students), my life has been enriched by the appreciation of the capital of his early generalizations of the Fibonacci sequence.

These attracted the attention of the Founding Editors of *The Fibonacci Quarterly*, Brother U. Alfred and Professor Verner E. Hoggatt, Jr. He accepted their invitation to become a foundation sustaining member and to join the initial board of Assistant Editors for Volume 1, Number 1, in February 1963.

Of that initial Board, Alwyn is still serving, together with Maxey Brooke, Paul Byrd, Leonard Carlitz, Henry Gould, and D. E. Thoro, but Alwyn is the only non-American still on it twenty-four years later. This is no mean feat and is indicative of his great virtue as a correspondent—the prompt reply—an asset which was to serve him well during his forty years on the staff of the University of New England where more than half the graduates have studied externally at a distance from UNE.

What follows is not a critical exposition of Alwyn's work—I have been too closely involved with him for twenty years as student, colleague, and friend. Nor is it an obituary—no one who knows Alwyn can expect an idle retirement. It is, in the words of the Editor's invitation, a list of his professional ac-complishments and a summary of some of his work.

2. CURRICULUM VITAE

Alwyn was born in 1923, son of a dairy farmer at Singleton, a small town in the Hunter Valley about 230 km north of Sydney, the capital of New South Wales. The original Horadams came from Wiesbaden in Germany in 1846 as "vine ten-derers." (The Hunter Valley is a renowned wine-producing area of Australia.)

It is of interest to note, in passing, that the other branch of the Horadam family emigrated to Texas, USA, and recently the two branches of the family have been in contact. A couple of years ago the Horadams had a family reunion on the original property, "Glendon," where there is a small church and ceme-tery.

Alwyn was educated at Maitland Boys' High School, which has a justifiably high reputation in Australia with many famous alumni. (Maitland is about 45 km southeast of Singleton, half way to the coastal city of Newcastle.) Alwyn dis-tinguished himself at school, where he was Dux, Captain of the School, Captain of the Criket team, and a member of the Football team.

In 1939, he went to the New England University College in Armidale at the other end of the Hunter Valley in the Northern Tablelands of the State. The

NEUC had only been founded the year before. It "was affiliated with the University of Sydney, and was Australia's first experiment in the establishment of a university in a purely country area—an experiment that had been urged by some educationists since the closing years of the nineteenth century" [25].

The New England region is one of the finest sheep-raising areas of Australia, and it is fitting that a university which was to become a world leader in Rural Science and Agricultural Economics began with the munificence of local pastoralists who donated a mansion and farms to secure its foundation. NEUC obtained its full autonomy in 1954, and it now attracts students from overseas as well as from every State of Australia. Its rapid expansion in recent years has not been at the expense of its rural setting: its beautiful campus is an attraction for academics who visit Australia.

Alwyn graduated as BA in 1942 at NEUC with first class honors awarded by the parent University of Sydney. During World War II, the University worked a four-term year, so he graduated in three years instead of four.

After graduating, he served as a school teacher with the NSW Department of Education. His studies included Mathematics, Education, and English. These, together with his work in schools, helped to make him not only a gifted teacher of mathematics but also a practitioner skilled in writing and research.

With further academic work, he completed the requirements for his Master of Arts, Diploma in Education, Doctor of Philosophy of the University of Sydney, and Bachelor of Education of the University of Melbourne. His Ph.D. was done under the supervision of Professor T. G. Room, FRS, a world-renowned geometer who was Head of the Department of Pure Mathematics at the University of Sydney for about thirty years. Alwyn's life-time interest in geometry culminated in the publication of his book on projective geometry [13]. His Ph.D. involved work with Clifford Matrices and showed the wide range of algebraic skills that he was later able to apply to number theory.

Alwyn joined the staff of NEUC in 1947 as a lecturer, and his role in the development of the University can be seen from the following list of his contributions to the university community. From the date of its independence in 1954 until 1972, he served as a Member of the University of New England's Governing Council. He was elected to this position by the University Convocation, that is, by the full university community of graduates and staff.

He has been Captain and President of the University Cricket Club, Vice-President of the University Football Club, President of the Science Society, President of the University Union, Foundation Secretary of the UNE Teacher's Association, Foundation Chairman of the UNE Alumni Association, and Foundation Fellow of Robb College (one of the residential colleges of UNE).

He also has been a Governor of Robb, Duval, and Wright Colleges, and the University Esquire Bedell. More recently, he has been the University Ombudsman, a difficult role and the appointment to which is an indication of the esteem of the university community for his integrity. He was also the UNE Delegate for the 13th Quinquennial Congress of British Commonwealth Universities in Birmingham (UK).

Considering all this involvement, it is almost a surprise to learn that he had time to get married in 1950. He and Mollie have now been together for 37 years and have three lovely daughters and two grand-daughters. Mollie is an engineer and number theorist with degrees BSc(Eng) (London), MA (Cambridge), Ph.D. (UNE), and, for many years was on the staff of UNE, which she continues to serve as a Member of Council and an Honorary Fellow in Mathematics.

The three daughters are all married. Kathryn, with a Ph.D. in mathematics from the Australian National University, is now on the staff of the Royal Melbourne Institute of Technology. Readers of *Mathematical Reviews* will thus have seen references to A. F., E. M., and K. J. Horadam at times. Actually, there

are five Dr. Horadams, because the other two daughters, Kerry and Alanna, have graduated as medical doctors!

Other academic appointments have included Dean of the Faculty of Science, and, in recent years, Head of the Department of Mathematics, Statistics and Computing Science, the position from which he retires. Is it any wonder that in correspondence, the Editor agreed that Alwyn is an excellent role model for an academic?

3. MATHEMATICS

In commenting on Alwyn's contributions to mathematics, I must first take account of his teaching ability, for that is how I first came to know him.

Since 1955, UNE has played a major role in distance-education through its Department of External Studies. Many school teachers of Mathematics took advantage of its facilities to upgrade their qualifications, and through them I had come to learn of Alwyn's expository skills. For this reason, I wrote to UNE in 1966 and I was fortunate to be assigned to Alwyn's care.

With undergraduates he aimed at the educational ideals of humane and liberal education through the medium of mathematics. With postgraduates, he encouraged optimism and a positive approach to research. I always found him cheerful but serious, able to ask the right questions and to resist the temptation to do too much for the novice researcher.

He encouraged his Master's and Doctoral research students, of whom he has successfully supervised 49, to correspond with mathematicians around the world to avoid insular or parochial frames of mind. His own research has been similarly stimulated with periods at the Universities of North Carolina, Cambridge, Leeds, Liverpool, East Anglia, Reading, York, Exeter, Iceland, and Malaya (where he was seconded to advise on their mathematics curriculum).

His influence on the teaching of mathematics at the high-school level in NSW has been threefold. First, as the NSW Universities' representative on the Board of Secondary Schools Studies Mathematics Syllabus Committee for the last 26 years. Second, as the co-author of a number of high school texts, of which [19] is an example. Finally, but not least significantly, through his help to high school mathematics teachers by his teaching of them, through his work for mathematics teaching associations, and through his writing of articles related to the teaching of mathematics.

His undergraduate expositions, which were clear and effective, would no doubt have, in turn, influenced the teaching styles of many of his proteges. Two of his short teaching texts for external students exemplify this. They also illustrate his interest in combinatorics. *Applied Combinatorics* [17] deals with graph theory, block designs, and enumeration techniques including recurrence relations and generating functions. Finite combinatorical structures and combinatorical circuits compose the two parts of his *Combinatorial Mathematics* [18]. He also co-authored a number of research papers which looked at combinatorial techniques for unravelling patterns (e.g., [21]).

To return briefly to his teaching texts. These are amply illustrated with worked examples and historical allusions. A more neutral observer, the eminent Oxford mathematician W. L. Ferrar, has noted in his review of Alwyn's *Outline Course of Pure Mathematics* [12]: "What a task!—and how well it has been carried out. The task? A unified treatment of the Algebra, Geometry and Calculus considered basic for the foundation of undergraduate mathematics.... Throughout, the author seizes every opportunity to interweave the variety of topics he is handling.... The range of knowledge and detailed reference displayed by the author is most striking.... That the author is an experienced teacher is everywhere apparent; he knows all the pitfalls" [1].

Alwyn's early research publications were in algebra and geometry [3-6, 8, 11] but the two which were seminal and continue to be cited frequently were in number theory. The second of these [10] generalized the Fibonacci numbers and introduced useful notation by considering the sequence

$$\{W_n\} \equiv \{W_n(a, b; p, q)\}$$

defined by the second-order linear homogeneous recurrence relation

$$W_n = pW_{n-1} - qW_{n-2}, \ n \geq 2,$$

with initial conditions $W_0 = a$, $W_1 = b$. Thus, the ordinary Fibonacci numbers $\{F_n\}$ are given by $\{W_n(0, 1; 1, -1)\}$. That paper, and a number which followed (e.g., [9]), developed the properties of this generalization, as he had done earlier for $\{H_n\}$ given by $\{W_n(a, b; 1, -1)\}$ [7]. These generalizations were not only elegant, they also clarified the roles of the fundamental and primordial sequences introduced by Lucas eighty odd years earlier [24].

By highlighting these two papers, I do not mean to do injustice to others, but they had a big influence on me, and they have been utilized by many others as well. The algebraic and geometric influences in Alwyn's research not surprisingly recur from time to time, as do techniques from the special functions of mathematical physics, especially the Chebyshev polynomials (e.g., [15]). Recently, he has co-authored material on the Gegenbauer polynomials and Gaussian Fibonacci numbers (e.g., [27]). There have also been numerous papers that deal with various properties of the Pell numbers $\{W_n(0, 1; 2, -1)\}$ and their generalizations and polynomials [22].

From time to time, too, there have been papers on other topics in number theory such as a proof of a problem posed by Morgan-Ward on the Staudt-Clausen theorem [23], Oresme numbers [14], and Wythoff pairs [16]. Surprisingly, since they were both number theorists, Mollie and Alwyn published only one paper together [20]. This dealt with finding the zeros of Fibonacci and Lucas polynomials and connections among them.

Among his many co-authors have been Stanley Collings (Open University, UK), Jamie Walton (Northern Rivers College), Brother Jim Mahon (Catholic·College of Education, Sydney), Peter Sekhon (NSWIT), Sharad Pethe (University of Malaya), Merv Dunkley (Macquarie University), I. W. Stewart (Mitchell College), Carl Chiarella (University of NSW), and Phil Loh (University of Sydney). Jamie, Jim, Peter, and the present writer are among his former Ph.D. students.

As well as the references cited, he has published in journals in Portugal, India, Argentina, and Malaysia. He has also presented papers at Conferences of the Australian Mathematical Society, the New Zealand Mathematics Colloquium, the Australian and New Zealand Association for the Advancement of Science, and as a guest speaker at numerous universities. As readers will be aware, he has also served on the International Committee of the First (Patras) and Second (San Jose) International Conferences on Fibonacci Numbers and Their Applications. He was also Co-Chairman of the latter.

4. CONCLUSION

If this article has only skimmed Alwyn's work, it is because the job could not be readily tackled in a brief space because he is the author (sole, joint) of 8 books and more than 90 papers. The task would be more appropriate to a Master's dissertation.

Nor have I gone into Alwyn's personal characteristics, to avoid him embarrassment. One, however, deserves mention, and it is his loyalty to people and to causes. In any case, I associate much of what is best in Alwyn with team sports such as cricket—at which he excelled, and which is not now held in high

regard by some educationists: "The old idea of a healthy mind in a healthy body, the benefit which physical training gives to the character by its discipline, the virtues of loyalty and self-abregation inculcated by team games, are all ideas which now evoke little but a faintly patronising tolerance or even contemptuous ridicule" [26].

I am very grateful for material supplied by Professor J. Hempel and Dr. E. M. Horadam of UNE to help me get started on this article. I apologize to readers and to Alwyn for any errors and inevitable omissions: it is difficult to write adequately about a living person without access to his files! I thank the Editor for the opportunity to honor a real university man, who has given much to mathematics internationally and, in a particular way, to the Fibonacci community.

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THIRD ANNUAL CONFERENCE
IS IN PLANNING STAGES

The third International Conference on Fibonacci Numbers and Their Applications is in the planning stages. Currently, it looks like the place will be Pisa, Italy, from July 25 to July 29, 1988.

More details will follow in the August 1987 issue. Plan now for another great conference.