$$t = \prod_{i=1}^{u} p_i, \quad 5 \le p_1 < p_2 < \cdots < p_u,$$

where $p_1, p_2, \ldots, p_u$ are primes. Then $p_2 \ge 7$, $p_3 \ge 11$, ... If $u \le 139$,

$$4 \le j = \frac{t-1}{\phi(t)} < \frac{t}{\phi(t)} = \prod_{i=1}^{u} \frac{p_i}{p_i - 1} \le \frac{5}{4}\,\frac{7}{6}\,\frac{11}{10} \cdots \frac{811}{810} < 4.$$

(There are 139 primes from 5 to 811, inclusive.) This contradiction shows that $u = \omega(t) \ge 140$ in this case, giving (iii) and completing the proof.

Using the above and results of Pomerance [6, esp. the Remark] and [7], it is not difficult to show that the number of natural numbers n such that $n \le x$, $(\phi(n) + 1)|n$ and n is not a prime or twice a prime, is

$$O\left(x^{1/2} (\log x)^{3\ 4} (\log \log x)^{-5/6}\right).$$

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