# ON THE GENERALIZED FIBONACCI PSEUDOPRIMES

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## 1. Introduction and Preliminaries

In this paper the results established by the first two authors in [3], [4], and [5] are extended and generalized.

After defining (in this section) classes of *generalized Lucas numbers*, $\{V_n(m)\}$, governed by the positive integral parameter $m$, the *Fibonacci pseudoprimes of the* $m^{\text{th}}$ *kind* ($m$-F.Psps.) are characterized in Section 2. A method for constructing them is discussed in Section 3, while some numerical results concerning these pseudoprimes are presented in Section 4. Finally, in Section 5, some possible further investigations in this field are outlined.

Let $m$ be an arbitrary natural number. The generalized Lucas numbers $V_n(m)$ (or simply $V_n$, if there is no fear of confusion) are defined (e.g., see [1] and [7]) by the second-order linear recurrence relation

(1.1) $\quad V_{n+2} = mV_{n+1} + V_n;\ V_0 = 2,\ V_1 = m.$

These numbers can also be expressed by means of the closed form expression (Binet's form)

(1.2) $\quad V_n = \alpha_m^n + \beta_m^n,$

where

$$(1.3)\quad \begin{cases} \Delta_m = \sqrt{m^2 + 4} \\ \alpha_m = (m + \Delta_m)/2 \\ \beta_m = -1/\alpha_m = (m - \Delta_m)/2. \end{cases}$$

It can be noted that, letting $m = 1$ in (1.1) and (1.2), the usual Lucas numbers $L_n$ are obtained.

The following *fundamental property* of the numbers $V_n$ has been established ([10], Eq. 108, p. 295): If $n$ is prime, then for all $m$,

(1.4) $\quad V_n(m) \equiv m \pmod{n}.$

## 2. The Fibonacci Pseudoprimes of the $m^{\text{th}}$ kind: Definition and Some Numerical Aspects

Rotkiewicz proved [15] that for each $m$, infinitely many odd composite numbers $n$ satisfy (1.4). Odd composite $n$ satisfying (1.4) are called *Fibonacci pseudoprimes of the* $m^{\text{th}}$ *kind* ($m$-F.Psps.). Write $s_k(m)$ for the $k^{\text{th}}$ one. Note that $s_1(1) = 705$, $s_1(2) = 169$, and $s_1(3) = 33$.

Some numerical aspects of the Fibonacci pseudoprimes of the $1^{\text{st}}$ kind [$s_k(1)$ or 1-F.Psps.] have been investigated by the authors in previous papers [3], [4], and [5]. In particular, we found that all 1-F.Psps. below $10^8$ are square-free and, as expected, most of them are congruent to 1 both modulo 4 (81.3%) and modulo 10 (63.2%). A heuristic argument to explain the popularity of the classes 1 modulo 4 and 1 modulo 10 can be constructed (cf. [12], p. 1018).

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Now, a question arises: "Do odd composites exist which are $m$-F.Psps. for distinct values of $m$?" The answer is the affirmative.

We define as *strong Fibonacci pseudoprimes of the $M^{\text{th}}$ kind* ($M$-sF.Psps.) all odd composites which are $m$-F.Psps. for $m = 1, 2, \ldots, M$. Obviously, from this definition, it follows that 1-F.Psps. and 1-sF.Psps. coincide and an $M$-sF.Psp. is an $m$-sF.Psp. ($1 \le m < M$) as well. For information, the smallest 2-sF.Psp. is $s_{14}(1) = 34{,}561$, while the smallest 3-sF.Psp. is $s_{89}(1) = 1{,}034{,}881$. Note that Theorem 6 of [4] states that a 1-F.Psp. is also a 4-F.Psp. so that all 3-sF.Psps. are also 4-sF.Psps.

A computer experiment was carried out [8] essentially to compile a table of 1-F.Psps. up to $10^8$ and to find $M$-sF.Psps. ($M > 1$) below this bound. The results can be summarized as follows. There are 852 1-F.Psps. below $10^8$ of which 48 are 2-sF.Psps. Four among these numbers are 4-sF.Psps. Among them, the rather exceptional number

$$s_{802}(1) = 87{,}318{,}001 = 17 \cdot 71 \cdot 73 \cdot 991$$

is a 7-sF.Psp. and is, at the same time, a *Carmichael number*. Carmichael numbers are composite numbers $n$ which satisfy the Fermat congruence $b^{n-1} \equiv 1 \pmod{n}$ for each $b$ relatively prime to $n$. Denoting the $k^{\text{th}}$ Carmichael number by $C_k$, we found that

$$s_{802}(1) = C_{244}.$$

## 2.1 Tables of 1-F.Psps: A Brief Historical Note

Earlier authors investigated the 1-F.Psps. and compiled tables of them up to certain bounds. To the best of our knowledge, apart from the sporadical discoveries of the first few 1-F.Psps. (e.g., see [11]; [5], Sec. 2), the oldest table (up to 555,200) containing, among other numbers, such pseudoprimes was compiled by Duparc [6] in 1955. In 1976 Yorinaga [17] compiled an analogous table to 707,000, and in 1983 Singmaster [16] published a table of 1-F.Psps. to 100,000 (these numbers were defined as *Lucas pseudoprimes* by the author). A table of 1-F.Psps. up to $10^6$ was given by the first two authors [5] in 1987.

The second author extended this table up to $10^8$ [8]. Copies of it will be sent, free of charge, upon request.

## 3. A Method To Obtain $m$-F.Psps.

In this section we offer a method to obtain generating formulas for the $m$-F.Psps. and, as a particular instance, we work out formulas for generating $M$-sF.Psps. ($M = 1, 2, 3, 4, 5$). The case $M = 1$ concerns, of course, numbers that are simply 1-F.Psps. Some numerical examples are also given.

First, let us state the following propositions.

*Proposition 1:* Let $p_i = 5k_i \pm 1$ and $q_j = 5h_j \pm 2$ be odd rational primes. Let

$$n = \prod_{i,j} p_i^a q_j^a \quad (a \in \{0, 1\})$$

be an odd composite and $\Lambda(n) = \operatorname{lcm}(p_i - 1, 2q_j + 2)_{i,j}$.

If $n - 1 \equiv 0 \,[\bmod\, \Lambda(n)]$, then $L_n \equiv 1 \pmod{n}$, that is, $n$ is a 1-F.Psp.

*Proposition 2:* If $p_i = 8k_i \pm 1$, $q_j = 8h_j \pm 3$, and $n - 1 \equiv 0 \,[\bmod\, \Lambda(n)]$, then $n$ is a 2-F.Psp.

*Proposition 3:* If $p_i = 13k_i \pm u$ ($u = 1, 3, 4$), $q_j = 13h_j \pm v$ ($v = 2, 5, 6$), and $n - 1 \equiv 0 \,[\bmod\, \Lambda(n)]$, then $n$ is a 3-F.Psp.

*Proposition 4:* If $p_i = 29k_i \pm u$ ($u = 1, 4, 5, 6, 9, 13$), $q_j = 29h_j \pm v$ ($v = 2, 3, 8, 10, 11, 12, 14$), and $n - 1 \equiv 0$ [mod $\Lambda(n)$], then $n$ is a 5-F.Psp.

*Proof of Proposition 1:* Since $\alpha_1$ and $\beta_1$, see (1.3), are integers (more precisely, unities) over the quadratic field $k(\sqrt{5})$, we have (see [9], p. 222)

$$(3.1) \qquad \alpha_1^{p_i - 1} \equiv \beta_1^{p_i - 1} \equiv 1 \pmod{p_i}$$

and (from [9], p. 223)

$$(3.2) \qquad \begin{cases} \alpha_1^{q_j + 1} \equiv N\alpha_1 = \alpha_1\beta_1 = -1 \pmod{q_j}, \\ \beta_1^{q_j + 1} \equiv N\beta_1 = \beta_1\alpha_1 = -1 \pmod{q_j}, \end{cases}$$

$N\xi$ being the norm of the element $\xi$ of a generic quadratic field.

If $n - 1 \equiv 0$ [mod $\Lambda(n)$], then by (3.1) we can write

$$(3.3) \qquad \alpha_1^{n-1} = \alpha_1^{t_i(p_i - 1)} = (\alpha_1^{p_i - 1})^{t_i} \equiv 1 \pmod{p_i} \quad (t_i \in \mathbb{N} = \{0, 1, 2, \ldots\})$$

and, analogously,

$$(3.4) \qquad \beta_1^{n-1} \equiv 1 \pmod{p_i}.$$

Under the same condition, by (3.2) we have

$$(3.5) \qquad \alpha_1^{n-1} = \alpha_1^{r_j(2q_j + 2)} = (\alpha_1^{q_j + 1})^{2r_j} \equiv 1 \pmod{q_j} \quad (r_j \in \mathbb{N})$$

and

$$(3.6) \qquad \beta_1^{n-1} \equiv 1 \pmod{q_j}.$$

Then, by (3.3)-(3.6) we obtain the congruences

$$(3.7) \qquad \alpha_1^n \equiv \alpha_1 \left(\bmod \prod_{i,j} p_i^a q_j^a\right) \quad (\text{i.e., mod } n)$$

and

$$(3.8) \qquad \beta_1^n \equiv \beta_1 \pmod{n}.$$

Finally, by (3.7) and (3.8) we have

$$L_n = \alpha_1^n + \beta_1^n \equiv \alpha_1 + \beta_1 = 1 \pmod{n}. \quad \text{Q.E.D.}$$

The proofs of Propositions 2, 3, and 4 are similar to that of Proposition 1 and are omitted for brevity.

## 3.1. Generating 1-F.Psps.

The first two examples offered in this subsection follow directly from Theorem 4 of [4] and give formulas for generating 1-F.Psps. which are, in addition, Carmichael numbers. The above mentioned theorem states that, if $n = p_1p_2\ldots p_s$, with $p_i$ a prime of the form $5k_i \pm 1$ ($1 \le i \le s$), is a Carmichael number, then $n$ is also a 1-F.Psp. Note that Proposition 1 generalizes this theorem.

*Example 1:* $n = p_1p_2p_3$

In 1939 Chernick invented *universal forms* for generating Carmichael numbers [2]. In this paper we refer to Ore's book [10] where these formulas are reported.

For constructing numbers $n$ of the above form (see [10], pp. 334-336), a suitable choice of the integral parameters $P_1$, $P_2$, and $P_3$ [*ibid.*] is necessary. For instance, for $P_1 = 5$, $P_2 = 1$, and $P_3 = 6$, we obtain

$$(3.9) \qquad n(t) = (30t + 19)(150t + 91)(180t + 109) \quad (t \in \mathbb{N}).$$

For all values of $t$ such that all three factors on the right-hand side of (3.9) are prime (necessarily of the form $5k_i \pm 1$), $n(t)$ is both a 1-F.Psp. and a Carmichael number. The smallest among such numbers is

$$n(4) = 79{,}624{,}621 = s_{766}(1) = C_{233}.$$

*Example 2:* $n = p_1p_2p_3p_4$

A formula yielding Carmichael numbers with four factors can be readily obtained from ([13], p. 99):

$$(3.10)\quad n(t) = (30t + 1)(60t + 1)(90t + 1)(180t + 1) \quad (t \in \mathbb{N}).$$

For all values of $t$ such that all four factors on the right-hand side of (3.10) are prime (necessarily of the form $5k_i \pm 1$), $n(t)$ is both a 1-F.Psp. and a Carmichael number. The smallest among such numbers is

$$n(9) = 192{,}739{,}365{,}541 = C_{4568}.$$

*Example 3:* $n = pq_1q_2$

The following example is based on Proposition 1. Let $p = 5k \pm 1$ and $q_j = 5h_j \pm 2$. It can be readily proved that, if $n - 1 \equiv 0$ [mod $\Lambda(n)$], then any two of the three numbers $p - 1$, $q_1 + 1$, and $q_2 + 1$ have the same greatest common divisor $d$. Therefore, we can write

$$(3.11)\quad p - 1 = dP,\ q_1 + 1 = dQ_1,\ q_2 + 1 = dQ_2$$

or

$$p - 1 = dP,\ 2q_1 + 2 = 2dQ_1,\ 2q_2 + 2 = 2dQ_2,$$

where the numbers $P$, $Q_1$, and $Q_2$ are relatively prime in pairs. Consequently, we have

$$\Lambda(n) = \text{lcm}(p - 1,\ 2q_1 + 2,\ 2q_2 + 2) = 2dPQ_1Q_2$$

and the sufficient condition for $n$ to be a 1-F.Psp. (see Proposition 1) takes the form

$$(3.12)\quad n = pq_1q_2 \equiv 1 \pmod{2dPQ_1Q_2}.$$

Following Ore (see [10], pp. 335-336), let us replace the values of $p$, $q_1$, and $q_2$ on the left-hand side of (3.12) by the corresponding values obtainable by (3.11). After some manipulations, omitted for brevity, we obtain the congruence

$$(3.13)\quad d(Q_1Q_2 - PQ_1 - PQ_2) + P - Q_1 - Q_2 \equiv 0 \pmod{2PQ_1Q_2}.$$

After choosing *suitable* values for $P$, $Q_1$, and $Q_2$, we find the smallest positive solution $d_0$ to the congruence (3.13) so that, by (3.11), we can write

$$(3.14)\quad \begin{cases} p = (d_0 + 2tPQ_1Q_2)P + 1, \\ q_1 = (d_0 + 2tPQ_1Q_2)Q_1 - 1, \quad (t \in \mathbb{N}) \\ q_2 = (d_0 + 2tPQ_1Q_2)Q_2 - 1. \end{cases}$$

The choice of $P$, $Q_1$, and $Q_2$ *must* yield a value of $d_0$ such that $p = 5k \pm 1$ and $q_j = 5h_j \pm 2$ $(j = 1, 2)$. For all values of $t$ such that all three numbers $p$, $q_1$, and $q_2$ are prime, $n$ is a 1-F.Psp. (but, in general, it is not a Carmichael number).

For instance, putting $P = 5$, $Q_1 = 1$, and $Q_2 = 2$ in (3.13), we obtain $d_0 = 14$ and, by (3.14),

$$(3.15)\quad n(t) = q_1q_2p = (20t + 13)(40t + 27)(100t + 71) \quad (t \in \mathbb{N}).$$

For $t \le 100{,}000$ there exist 641 1-F.Psps. of the above form. The smallest among them is

$$n(2) = s_{114}(1) = 1{,}536{,}841,$$

while the largest is

$$n(99{,}992) = 79{,}982{,}429{,}286{,}524{,}601{,}241.$$

Many more formulas for generating 1-F.Psps. can be obtained by means of other suitable choices of $P$, $Q_1$, and $Q_2$ in (3.13). As a further example, letting $P = 5$, $Q_1 = 2$, and $Q_2 = 9$, we get

$$(3.16) \quad n(t) = (360t + 203)(900t + 511)(1620t + 917) \quad (t \in \mathbb{N}).$$

For $t \le 100{,}000$ there exist 1255 1-F.Psps. of this form. The smallest among them is

$$n(10) = 619{,}127{,}589{,}961,$$

while the largest is

$$n(99{,}994) = 524{,}794{,}437{,}221{,}730{,}602{,}894{,}281.$$

It must be noted that the sets containing the 1-F.Psps. of the forms (3.15) and (3.16) are disjoint.

## 3.2 Generating $m$-F.Psps. ($m > 1$)

Using the results established in Section 3.1 and Propositions 2-4, we can derive formulas for generating $M$-sF.Psps. ($M$ = 2, 3, 4, 5).

For example, let us consider expression (3.15) which generates 1-F.Psps. and impose that $q_1$ (and $q_2$) and $p$ are of the forms $8h \pm 3$ and $8k \pm 1$, respectively (see Proposition 2). As a particular instance, if we impose that $p \equiv -1 \pmod 8$, then the congruence $t \equiv 0 \pmod 2$ must necessarily hold. For such values of $t$, the relations $q_1 = 8h_1 - 3$ and $q_2 = 8h_2 + 3$ turn out, so that the conditions of Proposition 2 are fulfilled (the congruence $n - 1 \equiv 0\ [\text{mod } \Lambda(n)]$ holds in (3.15), by construction).

Consequently, the numbers

$$\begin{aligned}(3.17) \quad n(t) = q_1 q_2 p &= (20 \cdot 2t + 13)(40 \cdot 2t + 27)(100 \cdot 2t + 71)\\ &= (40t + 13)(80t + 27)(200t + 71) \quad (t \in \mathbb{N})\end{aligned}$$

are 2-sF.Psps. for all values of $t$ such that all three factors on the right-hand side of (3.17) are prime. For $t \le 50{,}000$, there exist 329 2-sF.Psps. of this form. The smallest (largest) among them and the smallest (largest) 1-F.Psp. obtainable by (3.15) (for $t \le 100{,}000$) obviously coincide.

Analogously, by imposing the condition $p \equiv 3 \pmod{13}$ (see Proposition 3) to (3.17), we obtain the numbers

$$(3.18) \quad n(t) = (520t + 93)(1040t + 187)(2600t + 471) \quad (t \in \mathbb{N}),$$

which, for all values of $t$ such that all three factors are prime, are 3-sF.Psps. and, consequently (cf. the end of the fourth paragraph in Section 2), are also 4-sF.Psps. For $t \le 50{,}000$ there exist 256 such numbers. The smallest among them is

$$n(59) = 291{,}424{,}493{,}801{,}801,$$

while the largest is

$$n(49{,}976) = 175{,}508{,}922{,}783{,}506{,}139{,}921{,}721.$$

Finally, by imposing the condition $p \equiv -4 \pmod{29}$ (see Proposition 4) on (3.18), we obtain the numbers

(3.19) $n(t) = (15{,}080t + 2173)(30{,}160t + 4347)(75{,}400t + 10{,}871) \quad (t \in \mathbb{N})$

which, for all values of $t$ such that all three factors are prime, are 5-sF.Psps. For $t \le 25{,}000$ there exist 73 such numbers. The smallest among them is

$$n(47) = 3{,}593{,}246{,}900{,}779{,}046{,}281,$$

while the largest is

$$n(24{,}791) = 522{,}508{,}952{,}184{,}890{,}040{,}253{,}388{,}041.$$

It can be proved that numbers of the form (3.19) cannot be 6-F.Psps.

## 4. Carmichael Numbers and Generalized Fibonacci Pseudoprimes: A Computer Experiment

By means of this experiment, we sought numbers which are $M$-sF.Psps. for comparatively large $M$. Since the largest value of $M$ which we were aware of (namely, $M = 7$) pertains to a Carmichael number (namely, $C_{244} = 87{,}318{,}001$), we submitted all numbers $C_k < 25 \cdot 10^9$ to the test

(4.1) $V_{C_k}(m) \equiv m \pmod{C_k}$

for $m = 1, 2, 3, \ldots$, with the aid of an efficient computer algorithm which finds $V_n$ reduced modulo $n$ after $[\log_2 n]$ recursive calculations (cf. [14], pp. 114 ff.). We could carry out this experiment by virtue of the courtesy of the editor of this journal who placed the table of Carmichael numbers compiled by S. Wagstaff (Purdue University) (cf. [12]) at our disposal.

While this paper was being refereed, Professor Wilfrid Keller (Rechenzentrum der Universitaet Hamburg, FRG) kindly provided us with a table of all $C_k \le 10^{13}$ compiled by him. Submitting these numbers to the test (4.1) yielded the following update to the results obtained from Wagstaff's table.

There exist 19,278 Carmichael numbers below $10^{13}$:

| | |
|---|---|
| 3518 among them are 1-F.Psps. | 3518 among them are 1-sF.Psps. |
| 2767 are 2-F.Psps. | 599 are 2-sF.Psps. |
| 1735 are 3-F.Psps. | 63 are 3-sF.Psps. |
| 3679 are 4-F.Psps. | 63 are 4-sF.Psps. |
| 1104 are 5-F.Psps. | 9 are 5-sF.Psps. |
| 1643 are 6-F.Psps. | 8 are 6-sF.Psps. |
| 1258 are 7-F.Psps. | 4 are 7-sF.Psps. |
| 1307 are 8-F.Psps. | None of them is an 8-sF.Psp. |
| 1443 are 9-F.Psps. | |
| 1324 are 10-F.Psps. | |

The three additional 7-sF.Psps. we found are

$$C_{1092} = 3{,}998{,}554{,}561 = 31 \cdot 41 \cdot 199 \cdot 15{,}809,$$
$$C_{3662} = 103{,}964{,}580{,}721 = 37 \cdot 41 \cdot 43 \cdot 199 \cdot 8009,$$
$$C_{7122} = 669{,}923{,}876{,}161 = 17 \cdot 43 \cdot 97 \cdot 197 \cdot 199 \cdot 241.$$

Since none of these numbers is an 8-sF.Psp., the record was not beaten! We offered [4] a prize of 50,000 Italian lire to the first person who would communicate to us an 8-sF.Psp. (below $10^{100}$). Of course, at least one of its factors was also requested.

Currently, the smallest 8-sF.Psp. which we were able to construct [see Sec. 5(iv)] is the 29-digit Carmichael number

$$34{,}613{,}972{,}314{,}979{,}099{,}337{,}871{,}392{,}961$$

(three factors). Actually, this number is an 11-sF.Psp. The first author won the prize.

Incidentally, we used the above mentioned algorithm also to submit all composite Lucas numbers $L_p$ ($2 \le p \le 953$, $p$ either a prime or a power of 2) to the test

(4.2) $\quad V_{L_p}(m) \equiv m \pmod{L_p}$

for $m = 2$. We recall (see Corollaries 1 and 3 of [4]) that (4.2) holds for any $p$ if $m = 1$. The result of this experiment led us to formulate the following

*Conjecture 1:* No composite $L_p$ is a 2-F.Psp.

which implies the equivalent "$L_p$ is prime iff (4.2) holds for $m = 2$." If Conjecture 1 were proved, then a powerful tool for finding very large Lucas primes would have been discovered.

## 5. Future Work

The authors intend to continue their study on the properties of $m$-F.Psps. The principal aim of this further work is:

(i) to find the smallest $M$-sF.Psps. for $8 \le M \le 15$;

(ii) to evaluate the order of magnitude of the smallest $M$-sF.Psps. for $M > 15$;

(iii) to find the smallest $M$-sF.Psps. ($M > 2$) (if any) which are the product of exactly two distinct primes (the smallest 1-F.Psp. and 2-sF.Psp. of this form are $s_5(1) = F_{19} = 4181$ and $s_{202}(1) = 4{,}403{,}027$, respectively).

(iv) to establish formulas for generating $M$-sF.Psps. ($M \ge 2$) which are, at the same time, Carmichael numbers.

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# REPORT ON THE FOURTH INTERNATIONAL CONFERENCE ON FIBONACCI NUMBERS AND THEIR APPLICATIONS

Herta T. Freitag

Sponsored jointly by the Fibonacci Association and Wake Forest University, The Fourth International Conference on Fibonacci Numbers and Their Applications was held from July 30 to August 3, 1990. As the Conference took place at Wake Forest University, our foreign visitors especially gained a most enjoyable insight into one of America's delightful set-ups: a small, highly esteemed, liberal arts University, nestled at the outskirts of a faithfully restored eighteenth-century town, Winston-Salem, North Carolina.

Immediately upon arrival it became clear to us how carefully and competently—under the leadership of the co-chairmen of the International Committee, A. F. Horadam (Australia) and A. N. Philippou (Cyprus), as well as of the co-chairmen of the Local Committee, F. T. Howard and M. E. Waddill—our Conference had been planned and prepared. Special thanks must also go to G. E. Bergum, editor of our *Fibonacci Quarterly*, for arranging an outstanding program.

There were about 50 participants, 40 of whom presented papers. Of these, two were women. From some 13 different lands they came; beside the U.S., the host country, Italy would have won the prize for maximum attendance, then Canada and Scotland, closely followed by Australia and Japan.

Papers related to the Fibonacci numbers and their ramifications, and to recursive sequences and their generalizations, as well as those that analyzed and explained number relationships, were presented. Once again, as had been the case in our previous conferences, the diversity of the papers gave testimony to the fertility of Fibonacci-related mathematics, as well as to the fructification of ideas, brought about through our mutual but, at the same time, disparate interests. The interplay between theoretically oriented manuscripts and those that highlighted practical aspects was, again, conspicuous and fascinating.

The Conference was held in the new Olin Physical Laboratory, which was accessible via overcoming several road hurdles that were necessitated by construction work across the campus. Although our hosts were most apologetic about this, we saw it as a sign of a vital, dynamic and, indeed, growing University.

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