Hence for $A = 6$, $B = 1/10$ it follows that

$$C'_{n+2} - C'_{n+1} - C'_n = F_{n+2} \quad .$$

Clearly

$$C_n = n/2\ F_n + n/10\ L_n + aF_n + bL_n \quad .$$

Taking $n = 0$ we get $b = 0$. For $n = 1$ we get $a = 2/3$. Therefore we have

$$C_n = n/2\ F_n + n/10\ L_n + 2/5\ F_n = \frac{n\ L_{n+1} + 2F_n}{5} \quad .$$

Also solved by Ronald Weimshenk, John L. Brown, Jr., Donald Knuth, H.H. Ferns and the proposer.

Editorial Note: Another characterization, besides the convolution

$$C_{n+1} = \sum_{r=1}^{n+1} F_r F_{n-r} = \frac{(n+1)L_{n+2} + 2F_{n+1}}{5} \quad ,$$

is the number of crossings of the interface, in the optical stack in problem B-6, Dec. 1963, p. 75, for all rays which are reflected n-times. If $f_0(x) = 0$, $f_1(x) = 1$, and $f_{n+2}(x) = xf_{n+1}(x) + f_n(x)$, the Fibonacci polynomials, then

$$f_n(1) = F_n \text{ and } f'_n(1) = C_{n-1} \quad .$$

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MATH MORALS

Brother U. Alfred

A tutor who tutored two rabbits,
Was intent on reforming their habits.
Said the two to the tutor,
"There are rabbits much cuter,
But non-Fibonacci, dagnabits."*

*The author has just taken out poetic license $\#F_{97}$ according to one clause of which it is permissible to corrupt corrupted words.

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