2. What is the limiting density of $\{U_r\}$ in the positive integers?

Similar results are found when one considers "Lucas Nim" analogous to Fibonacci Nim.

REFERENCES

1. Brother U. Alfred, "Research Project: Fibonacci Nim," Fibonacci Quarterly, 1(1963), No. 1, p. 63.
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ON SUMS $F_x^2 \pm F_y^2$

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Formulas for the sum of the squares of Fibonacci numbers are:

(1) $$F_{n+2k}^2 + F_n^2 = F_{n+2k-2}F_{n+2k+1} + F_{2k-1}F_{2n+2k-1}$$

(2) $$F_{n+2k+1}^2 + F_n^2 = F_{2k+1}F_{2n+2k+1}$$

(3) $$F_{n+2k}^2 - F_n^2 = F_{2k}F_{2n+2k}$$

(4) $$F_{n+2k+1}^2 - F_n^2 = F_{n-1}F_{n+2} + F_{2k}F_{2n+2k+2}$$

Validity of the above is established by using:

$$F_n = \frac{1}{\sqrt{5}}(\alpha^n - \beta^n),\ L_n = \alpha^n + \beta^n,\ \alpha = \frac{1+\sqrt{5}}{2},\ \beta = \frac{1-\sqrt{5}}{2},\ \alpha\beta = -1\ .$$

For example:

$$5(F_{n+2k+1}^2 - F_n^2) =$$

$$(\alpha^{2n+4k+2} + \beta^{2n+4k+2}) - (\alpha^{2n} + \beta^{2n}) - 2\alpha^n\beta^n(\alpha^{2k+1}\beta^{2k+1} - 1) =$$

$$L_{2n+4k+2} - L_{2n} - 2(-1)^n(-2) = L_{2n+4k+2} - L_{2n} - (-1)^{n-1}L_3$$

$$5(F_{n-1}F_{n+2} + F_{2k}F_{2n+2k+2}) =$$

$$(\alpha^{2n+1} + \beta^{2n+1}) + (\alpha^{2n+4k+2} + \beta^{2n+4k+2}) - \alpha^{n-1}\beta^{n-1}(\alpha^3+\beta^3) -$$

$$- \alpha^{2k}\beta^{2k}(\alpha^{2n+2}+\beta^{2n+2}) =$$

$$L_{2n+4k+2} + (L_{2n+1} - L_{2n+2}) - (-1)^{n-1}L_3 = L_{2n+4k+2} - L_{2n} - (-1)^{n-1}L_3.$$

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