REPLY TO EXPLORING FIBONACCI MAGIC SQUARES*

JOHN L. BROWN, JR.
Pennsylvania State University, State College, Pennsylvania

Problem. For $n \geq 2$, show that there do not exist any nxn magic squares with distinct entries chosen from the set of Fibonacci numbers, $u_1 = 1$, $u_2 = 2$, $u_{n+2} = u_{n+1} + u_n$ for $n \geq 1$.

Proof. Trivial for $n = 2$.

If an nxn magic square existed for some $n \geq 3$ with distinct Fibonacci entries, then the requirement that the first three columns add to the same number would yield the equalities:

$$(*)\quad F_{i_1} + F_{i_2} + \ldots + F_{i_n} = F_{j_1} + F_{j_2} + \ldots + F_{j_n} = F_{k_1} + F_{k_2} + \ldots + F_{k_n} .$$

Since the entries are distinct, we may assume without loss of generality that $F_{i_1} > F_{i_2} > \ldots > F_{i_n}$, $F_{j_1} > F_{j_2} > \ldots > F_{j_n}$ and $F_{k_1} > F_{k_2} > \ldots > F_{k_n}$.

Noting that the columns contain no common elements, and by rearrangement if necessary, we assume $F_{i_1} > F_{j_1} > F_{k_1}$, again without losing generality; thus, $F_{i_1} \geq F_{k_1 + 2}$.

Now

$$F_{i_1} + F_{i_2} + \ldots + F_{i_n} > F_{i_1} \geq F_{k_1+2} ,$$

while

$$F_{k_1} + F_{k_2} + \ldots + F_{k_n} \leq \sum_{1}^{k_1} F_i = F_{k_1+2} - 1 .$$

This contradicts the equality postulated in (*), and we conclude no magic squares in distinct Fibonacci numbers are possible.

*The Fibonacci Quarterly, October 1964, Page 216.

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