A RECURSIVE OPERATION ON TWO-DIGIT INTEGERS

CHARLES W. TRIGG San Diego, California

Let d be the sum of the digits of the integer S, and $S_1 + d_1 = S_2$, $S_2 + d_2 = S_3$, ..., $S_k + d_k = S_{k+1}$, with each S being reduced modulo 100. Since there are only 100 two-digit integers (with initial zeros being permitted), this operation must eventually repeat one or more integers with the production of one or more closures of the series into loops. Indeed, the 100 integers fall into four groups of branched chains of 2, 11, 25 and 62 integers terminating in loops of 1, 11, 13 and 20 members, respectively. The loops consist of the integers between the asterisks (**) in the following series.

- (1) 86*00*00
- (2) *52 59 73 83 94 07 14 19 29 40 44*52
- (3) 20 22 26 34 41 46 56 67 80 88 75 87*02 04 08 16 23 28

38 49 62 70 77 91 01*02

37 47 58 71 79 95 09 18 27 36 45 54 63 72 81 90 99*17.

Thus all the numbers can be generated from the eleven integers 86, 52, 20, 75, 64, 31, 53, 24, 42, 97 and 47.

If the S's are reduced modulo 10, two groups are formed — a loop of one with one attachment, 5*0*0, and a loop of four with four attachments, $\frac{1-2-4-7}{3-6-8-9}$, so that it takes five digits to generate the set.