



# The Fibonacci Quarterly

196

THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION

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VOLUME 33

FEBRUARY 1995

NUMBER 1

## PURPOSE

The primary function of **THE FIBONACCI QUARTERLY** is to serve as a focal point for widespread interest in the Fibonacci and related numbers, especially with respect to new results, research proposals, challenging problems, and innovative proofs of old ideas.

## EDITORIAL POLICY

**THE FIBONACCI QUARTERLY** seeks articles that are intelligible yet stimulating to its readers, most of whom are university teachers and students. These articles should be lively and well motivated, with new ideas that develop enthusiasm for number sequences or the exploration of number facts. Illustrations and tables should be wisely used to clarify the ideas of the manuscript. Unanswered questions are encouraged, and a complete list of references is absolutely necessary.

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All back issues of **THE FIBONACCI QUARTERLY** are available in microfilm or hard copy format from **UNIVERSITY MICROFILMS INTERNATIONAL, 300 NORTH ZEEB ROAD, DEPT. P.R., ANN ARBOR, MI 48106.** Reprints can also be purchased from **UMI CLEARING HOUSE** at the same address.

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# *The Fibonacci Quarterly*

*Founded in 1963 by Verner E. Hoggatt, Jr. (1921-1980)  
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*THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION  
DEVOTED TO THE STUDY  
OF INTEGERS WITH SPECIAL PROPERTIES*

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