NOTE ON THE EUCLIDEAN ALGORITHM

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Let $\mu_1 = 1$; $\mu_2 = 2$ and $\mu_n = \mu_{n-1} + \mu_{n-2}$ $(n \geq 3)$ be the Fibonacci numbers. Then the Euclidean Algorithm for finding (μ_{n+1}, μ_n) is

$$\begin{aligned} \mu_{n+1} &= \mu_n + \mu_{n-1} \\ \mu_n &= \mu_{n-1} + \mu_{n-2} \\ &\;\;\vdots \\ \mu_3 &= \mu_2 + \mu_1 \\ \mu_2 &= 2\mu_1 \end{aligned}$$

and the required number of divisions is n.

Let

$$\xi = \frac{1 + \sqrt{5}}{2} \quad .$$

Then $\xi > \mu_1$, and, since ξ is a root of the equation $\xi^2 = \xi + 1$, we have $\xi^2 > \mu_2$. Also, $\xi^3 = \xi^2 + \xi > \mu_2 + \mu_1 = \mu_3$ and, in general, $\xi^n > \mu_n$.

Now let p be the number of digits in μ_n. Then $\mu_n \geq 10^{p-1}$ and, by the preceding result, $10^{p-1} < \xi^n$ or

$$n > \frac{p - 1}{\log \xi} \quad .$$

Hence

$$n > \frac{p}{\log \xi} - 5 \quad \text{since} \quad \log \xi > \frac{1}{5}$$

In the proof of Lamé's Theorem [1] it is shown that

$$n < \frac{p}{\log \xi} + 1 \quad ,$$

where here n is the number of divisions in the Euclidean Algorithm for any two numbers and p is the number of digits in the smaller number. Thus we see that the upper bound for the number of divisions in the Euclidean Algorithm given by Lamé's Theorem is virtually the best possible.

REFERENCE

1. J. V. Uspensky and M. A. Heaslet, Elementary Number Theory, McGraw-Hill, 1939, p. 43.

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