



# The Fibonacci Quarterly

THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION

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## PURPOSE

The primary function of **THE FIBONACCI QUARTERLY** is to serve as a focal point for widespread interest in the Fibonacci and related numbers, especially with respect to new results, research proposals, challenging problems, and innovative proofs of old ideas.

## EDITORIAL POLICY

**THE FIBONACCI QUARTERLY** seeks articles that are intelligible yet stimulating to its readers, most of whom are university teachers and students. These articles should be lively and well motivated, with new ideas that develop enthusiasm for number sequences or the exploration of number facts. Illustrations and tables should be wisely used to clarify the ideas of the manuscript. Unanswered questions are encouraged, and a complete list of references is absolutely necessary.

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Articles should be submitted using the format of articles in any current issues of **THE FIBONACCI QUARTERLY**. They should be typewritten or reproduced typewritten copies, that are clearly readable, double spaced with wide margins and on only one side of the paper. The full name and address of the author must appear at the beginning of the paper directly under the title. Illustrations should be carefully drawn in India ink on separate sheets of bond paper or vellum, approximately twice the size they are to appear in print. Since the Fibonacci Association has adopted  $F_1 = F_2 = 1$ ,  $F_n + 1 = F_n + F_{n-1}$ ,  $n \geq 2$  and  $L_1 = 1$ ,  $L_2 = 3$ ,  $L_n + 1 = L_n + L_{n-1}$ ,  $n \geq 2$  as the standard definitions for The Fibonacci and Lucas sequences, these definitions *should not* be a part of future papers. However, the notations *must* be used. One to three *complete* A.M.S. classification numbers *must* be given directly after references or on the bottom of the last page. **Papers not satisfying all of these criteria will be returned.** See the worldwide web page at:

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# The Fibonacci Quarterly

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DEVOTED TO THE STUDY OF INTEGERS WITH SPECIAL PROPERTIES

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