



# The Fibonacci Quarterly

THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION

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## PURPOSE

The primary function of **THE FIBONACCI QUARTERLY** is to serve as a focal point for wide-spread interest in the Fibonacci and related numbers, especially with respect to new results, research proposals, challenging problems, and innovative proofs of old ideas.

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# The Fibonacci Quarterly

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DEVOTED TO THE STUDY OF INTEGERS WITH SPECIAL PROPERTIES

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