A GENERAL FIBONACCI FUNCTION

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Probably many of us who have an interest in Fibonacci series have plotted F_n as a function of n on graph paper. If we connect the points with straight line segments on cartesian coordinate paper, we achieve a continuous piecewise linear Fibonacci Function (see Fig. 1).

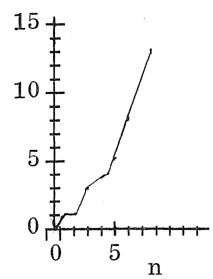

Fig. 1 The Fibonacci Function

This Fibonacci Function has many interesting properties other than at the integral values of the n. In fact, this function gives rise to the concept of F_x, where x is any real number.

If we tabulate the function, it becomes easier to discern the relationships involved.

PARTIAL TABLE OF THE FIBONACCI FUNCTION
F_x Versus x (tenths)

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	Δ
0	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	.1
1	1	1	1	1	1	1	1	1	1	1	0
2	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	.1
3	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	.1
4	3	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	.2
5	5	5.3	5.6	5.9	6.2	6.5	6.8	7.1	7.4	7.7	.3
6	8	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	.5
7	13	13.8	14.6	15.4	16.2	17.0	17.8	18.6	19.4	20.2	.8

(Example: $F_{6.3} = 9.5$)

One immediately notes that between $x = 0$ and $x = 1$, $F_x = x$. Because of this, it is convenient to set

$$x = n + r ,$$

where n is an integer and r is the balance less than unity. Thus:

$$F_r = r$$
$$F_{1+r} = 1$$
$$F_{2+r} = 1 + r$$
$$F_{3+r} = 2 + r$$
$$F_{4+r} = 3 + 2r$$
$$F_{n+r} = F_n + F_{n-1}r$$
$$F_x = F_n + F_{n-1}\, r$$

One may also observe in any column in the table, that any particular entry is the sum of the preceding two entries, i. e.,

$$F_{x+1} = F_x + F_{x-1}$$

Other interesting properties that are obvious by inspection include:

$$2\,F_{n+0.5} = F_{n+2}$$
$$3\,F_{n+0.333} = L_{n+1} ,$$ where L is the Lucas number.

Not so obvious is the fact that there are relationships between the squares and the products of the entries in any column of the table. In fact

$$F_x^2 = \left[(F_{x-1})(F_{x+1}) - 1\right] + r^2 + r$$

and when r is the golden ratio (0.618034)

$$F_{x\,\text{golden}}^2 = (F_{x-1})(F_{x+1})$$

The proof is left to the reader.

Note also that this function allows any Fibonacci-type sequence to be normalized into the $r, 1, 1+r$ form. For example, a $2, 10, 12, 22\cdots$ sequence converts to a $0.2, 1\cdots$ general type sequence by dividing by 10.

CONCLUSION

In general, this particular method of expressing the Fibonacci Function has the potential of being a rich area of Fibonacci discovery. Possibilities include verification and reformulation of all Fibonacci formulae. Also an inverse table of F_x's versus all the real numbers may be formed and investigated.

Because this function represents the normalization of all Fibonacci-type sequences, any results should demonstrate broad fulfillment of the goals of the investigator.

REFERENCE

1. Dewey C. Duncan, "Chains of Fibonacci-Wise Triangles," Fibonacci Quarterly Journal, Feb. 1967.

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