CURIOSA IN 1967

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(A)
$$\begin{aligned}1967 &= (-1 + 9 + 6 - 7)(196 - 7 + 91 - 6 + 7)\\ &= -12 + (34)(56) + 78 - \sqrt{9}\\ &= 0! + 1! + 2(3!)(4!) + (5)(6)(7)(8) - \sqrt{9}\\ &= 2^0 + 2^1 + 2^2 + 2^3 + 2^5 + 2^7 + 2^8 + 2^9 + 2^{10}\end{aligned}$$

(B)
$$\begin{aligned}1967_{10} &= 117E_{12} = 1529_{11} = 2625_9 = 3657_8 = 5510_7 = 13035_6\\ &= 30332_5 = 132233_4 = 2200212_3\\ &= 11110101111_2, \text{ a palindrome.}\end{aligned}$$

(C) $(1!\,9!\,6!\,7!)(!1!\,9!\,6!\,7!) = 0$, where $!x$ is subfactorial x.

(D) Expressed in Fibonacci numbers:
$$\begin{aligned}1967 &= 1 - 8 + 377 + 1597\\ &= 1597 + 377 - 5 - 2\\ &= 1 + 13 + 34 + 89 + 233 + 1597\\ &= 1 + 2 + 3 + 8 + 13 + 21 + 34 + 55 + 89 + 144 + 233 + 377 + 987\end{aligned}$$

(E) Four squares can be formed from the digits of 1967, namely: 196, 169, 961, and 16, which latter also is a fourth power.

$$\begin{aligned}1967 &= (4^2 - 3^2)(5^2 + 16^2)\\ &= 144^2 - 137^2 = 984^2 - 983^2\\ &= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 8^2 + 9^2 + 10^2 + 11^2 + 12^2\\ &\quad + 14^2 + 15^2 + 16^2 + 17^2 + 20^2\end{aligned}$$

(F)
$$\begin{aligned}1967 &= (1111 - 111 - 11)(1 + 1) - 11\\ &= 22^2 \cdot 2^2 + 2^{2+2+2/2} - 2/2\\ &= (3 + 3)(333) - 33 + 3 - 3/3\\ &= 44^{\sqrt{4}} + 4(4 + \sqrt{4}) - 4/4\\ &= (555 - 5/5)(5 - 5/5) - 5(55 - 5) + 5/5\\ &= (666 + 6/6)(6 + 6 + 6)/6 - 6(6) + (6 + 6)/6\end{aligned}$$

$$= 777 + 77(7 + 7) + (777 + 7)/7$$
$$= (888 + 88 + 8 - 8/8)(8 + 8)/8 + 8/8$$
$$= (999 - \sqrt{9})(9 + 9)/9 - 9\sqrt{9} + (9 + 9)/9$$

(G) Here are several ways in which 1967 can be written using conventional mathematical symbols and one 1, nine 9's, six 6's, and seven 7's in order from left to right.

$$1967 = 19(99 + 999/999) + 66 + 66/66 + 7(777 - 777)$$
$$= 19(99 + 999/999) + 66(66/66) + 7^{(777-777)}$$
$$= 19(99 + 999/999) + 6(66/6 - 6/6) + 7(777/777)$$
$$= 1(999 + 9/9) + 9(99 + \sqrt{9} - 6/6 - 6/6)$$
$$+ 6(6 - 7 + 77/7) + 7(7/7)$$

(H) If to 1967 its reversal is added, and the process repeated several times, a palindromic number is produced in five operations.

$$\begin{array}{r} 1967 \\ \underline{7691} \\ 9658 \\ \underline{8569} \\ 18227 \\ \underline{72281} \\ 90508 \\ \underline{80509} \\ 171017 \\ \underline{710171} \\ 881188 \end{array}$$

(I) 7691 - 1967 = 5724, 5724 - 4275 = 1449, 9441 - 1449 = 7992, 7992 - 2997 = 4995, 5994 - 4995 = 99, a palindromic number after five subtractions.

(J) If the digits of 1967 be written in descending order before reversal and subtraction and the process be repeated continuously:

9761 - 1679 = 8082, 8820 - 0288 = 8532,

8532 - 2358 = 6174, 7641 - 1467 = 6174,

Thus Kaprekar's constant 6174 is reached in three operations.

(K)

The circulant $\begin{vmatrix} 1 & 9 & 6 & 7 \\ 7 & 1 & 9 & 6 \\ 6 & 7 & 1 & 9 \\ 9 & 6 & 7 & 1 \end{vmatrix} = -3^2(23)(29)$.

$$\begin{vmatrix} 1 & 7 \\ 6 & 9 \end{vmatrix} \text{ divides } \begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 6 & 7 & 6 \\ 6 & 7 & 6 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix}, \text{ that is, } \frac{3(11)^2}{-3(11)} = -11 .$$

$$\begin{vmatrix} 1 & 9 \\ 7 & 6 \end{vmatrix} \text{ divides } \begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 1 & 1 & 6 \\ 6 & 1 & 1 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix}, \text{ that is, } \frac{9(11)(19)}{-3(19)} = -33 .$$

$$\begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 9 & 9 & 6 \\ 6 & 9 & 9 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix} = 9^3. \quad \begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 6 & 6 & 6 \\ 6 & 6 & 6 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix} = 3^3(43) . \quad \begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 7 & 7 & 6 \\ 6 & 7 & 7 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix} = 3^2(113) .$$

$$\begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 0 & 0 & 6 \\ 6 & 0 & 0 & 9 \\ 7 & 6 & 9 & 1 \end{vmatrix} = 45^2 . \quad \begin{vmatrix} 1 & 9 & 6 & 7 \\ 9 & 6 & 7 & 0 \\ 6 & 7 & 0 & 0 \\ 7 & 0 & 0 & 0 \end{vmatrix} = 7^4 .$$

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(Continued from p. 473.)

Of the twenty-four four-digit numbers that can be written with the digits of 1967, seven are prime:

1697, 6197, 6719, 6791, 6917, 6971, and 7691 .

(F)

$$-\begin{vmatrix} 1 & 9 \\ 6 & 7 \end{vmatrix} = 47 .$$

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