$$\begin{split} \mathbf{F}_{3\mathbf{n}} &= \mathbf{F}_{\mathbf{n}} \mathbf{F}_{2\mathbf{n}-1} + \mathbf{F}_{2\mathbf{n}} \mathbf{F}_{\mathbf{n}+1} \\ &= \mathbf{F}_{\mathbf{n}} (\mathbf{F}_{\mathbf{n}-1}^2 + \mathbf{F}_{\mathbf{n}}^2) + (\mathbf{F}_{\mathbf{n}} \mathbf{F}_{\mathbf{n}-1} + \mathbf{F}_{\mathbf{n}} \mathbf{F}_{\mathbf{n}+1}) \mathbf{F}_{\mathbf{n}+1} \\ &= \mathbf{F}_{\mathbf{n}}^3 + \mathbf{F}_{\mathbf{n}} \mathbf{F}_{\mathbf{n}+1}^2 + \mathbf{F}_{\mathbf{n}-1} \mathbf{F}_{\mathbf{n}} (\mathbf{F}_{\mathbf{n}+1} + \mathbf{F}_{\mathbf{n}-1}) \\ &= \mathbf{F}_{\mathbf{n}}^3 + \mathbf{F}_{\mathbf{n}} (\mathbf{F}_{\mathbf{n}}^2 + 2\mathbf{F}_{\mathbf{n}} \mathbf{F}_{\mathbf{n}-1} + \mathbf{F}_{\mathbf{n}-1}^2) + \mathbf{F}_{\mathbf{n}-1} \mathbf{F}_{\mathbf{n}} (\mathbf{F}_{\mathbf{n}+1} + \mathbf{F}_{\mathbf{n}-1}) \\ &= 2\mathbf{F}_{\mathbf{n}}^3 + 2\mathbf{F}_{\mathbf{n}-1} \mathbf{F}_{\mathbf{n}} (\mathbf{F}_{\mathbf{n}-1} + \mathbf{F}_{\mathbf{n}}) + \mathbf{F}_{\mathbf{n}-1} \mathbf{F}_{\mathbf{n}} \mathbf{F}_{\mathbf{n}+1} \\ &= 2\mathbf{F}_{\mathbf{n}}^3 + 3\mathbf{F}_{\mathbf{n}-1} \mathbf{F}_{\mathbf{n}} \mathbf{F}_{\mathbf{n}+1} \end{split}$$

Substituting this in (1) we get

$$I = L_k F_k^2 F_{3n} + (-1)^k F_n^3 L_k$$

Therefore,

$$F_{n+k}^3 + (-1)^k F_{n-k}^3 = L_k \left[F_k^2 F_{3n} + (-1)^k F_n^3 \right]$$

Also solved by Charles R. Wall.

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(Continued from p. 138.)

All known Fibonacci equations using $\,F_n^{}\,$ are theoretically generalizable to equations using $\,F_{_X}^{}\,$. For some examples, see [2]. See [3] also.

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