Case 2. $k = 1$. Note that

$$\varphi(n) = \varphi(2)\varphi(s) = \varphi(s) = \varphi\left(\frac{n}{2}\right) ,$$

so

$$\varphi(T_n) = \varphi\left(\frac{n}{2}\right)\varphi(n + 1) = \varphi(n)\varphi(n + 1) .$$

Case 3. $k > 1$. Now

$$\varphi(n) = \varphi(2^k)\varphi(s) = 2^{k-1}\varphi(s) ,$$

and

$$\varphi\left(\frac{n}{2}\right) = 2^{k-2}\varphi(s) .$$

Also we obviously have $n + 1 > 2$; so let $\varphi(n + 1) = 2m$, where m is an integer. Then

$$\varphi(T_n) = \varphi\left(\frac{n}{2}\right)\varphi(n + 1) = 2^{k-2}\varphi(s)2m = m \cdot 2^{k-1}\varphi(s) = m\varphi(n) .$$

Also solved by Herta T. Freitag, Guy A. Guillottee (Canada), Serge Hamelin (Canada), Douglas Lind (England), C. B. A. Peck, Gregory Wulczyn, and the Proposer.

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[Continued from page 538.]

or

$$491 = x(x + 1) + y(y + 1) + z(z + 1) .$$

This is impossible, since $x(x + 1)$, $y(y + 1)$, and $z(z + 1)$ are all even.

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