$$b(2k + 1) = b(k + 1) + b(k) \quad \text{for} \quad k \geq 1 .$$

For $n \geq 1$, show the following:

(a) $$b([2^{n+1} + (-1)^n /3) = F_{n+1} .$$

(b) $$b([7 \cdot 2^{n-1} + (-1)^n]/3) = L_n .$$

Solution by Michael Yoder, Student, Albuquerque Academy, Albuquerque, New Mexico.

(a) For $n = 0,1$ the formula is easily verified. Assume it is true for $n - 2$ and $n - 1$ with $n \geq 2$; then if n is even,

$$\begin{aligned} b[(2^{n+1} + 1)/3] &= b[(2^n - 1)/3 + b\ (2^n + 2)/3] \\ &= F_n + b[(2^{n-1} + 1)/3] \\ &= F_n + F_{n-1} = F_{n+1} . \end{aligned}$$

Similarly, if n is odd,

$$b[(2^{n+1} - 1/3] = F_{n+1} .$$

(b) For $n = 1,2$ the theorem is true; and by exactly the same argument as in (a), it follows by induction for all positive integers n.

Also solved by Herta T. Freitag and the Proposer.

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(Continued from page 101.)

SOLUTIONS TO PROBLEMS

1. $5n^3 - 4n^2 + 3n - 8$.
2. $3n^2 - 8n + 4$ and the Fibonacci sequence: $1,4,5,9,14,\cdots$.
3. $7n^3 + 3n^2 - 5n + 2 + 3x2^n$.
4. $4n + 3 + 3(-1)^n$.
5. $2n^3 - 3n^2 - n + 5$ and the Fibonacci sequence $4L_n$.

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