FIBONACCI SERIES IN THE SOLAR SYSTEM

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ABSTRACT

The Fibonacci Series is shown to predict the distances of the moons of Jupiter, Saturn and Uranus from their respective primary. The planets are shown to have a trend which follows the Fibonacci Series with individual offsets attributed to planetary densities.

1. INTRODUCTION

Many series exist where successive terms are a function of previous terms. When this function is a linear combination, each term can be expressed as

$$z_n = \sum_{j=1}^{n} a_j z_{n-j} + C$$

where $\{a_j\}$ is a weighting function acting upon each successive $\{z_{n-j}\}$, and C is a constant.

The Fibonacci series $\{f_n\} = \{0, 1, 1, 2, 3, 5, 8, 13, \cdots\}$ which is of the form

(1) $$f_n = f_{n-1} + f_{n-2}$$

is an excellent example of this with $\{a_j\} = \{1, 1, 0, \cdots, 0\}$ and $C = 0$.

Subsequently, however, a modified Fibonacci Series will be employed having the form $\{1, 2, 3, 5, \cdots\}$ but with again Eq. (1) holding.

2. PLANET MOONS

The distances of moons from their parent planet have not been known to follow any sequence. The mean distances z_n of the moons of Jupiter, Saturn, and Uranus, as shown in Figs. 1 through 5, respectively, can be assigned to terms in the Fibonacci Series f_n in the form

(2) $$z_n = mf_n + b \;.$$

The following table shows the "best fit" values of m and b for Jupiter, Saturn, and Uranus and the possible values for Mars and Neptune which, because they have only two moons each, cannot be fitted to a linear relationship.

Table: Values of m and b

Primary	No. of Moons	m (10^3 km)	b (10^3 km)
Mars	2	14	-4.8
		7	2.3
		3.5	5.8
Jupiter	12	240	-50
Saturn	9	50	140
Uranus	5	70	60
Neptune	2	420	-66
		250	104

Equation (2) is expressible in the form

$$z_n = z_{n-1} + z_{n-2} - b \;,$$

comparable to Eq. (1).

3. PLANETS

Figure 6 shows the mean distance of the planets from the sun, z_n, plotted against the Fibonacci Series f_n. As can be seen, the trend of the distances follow the series, although the individual values seem to be offset in a sinusoidal manner from it. Figure 7, however, shows the quantity z_n/f_n for

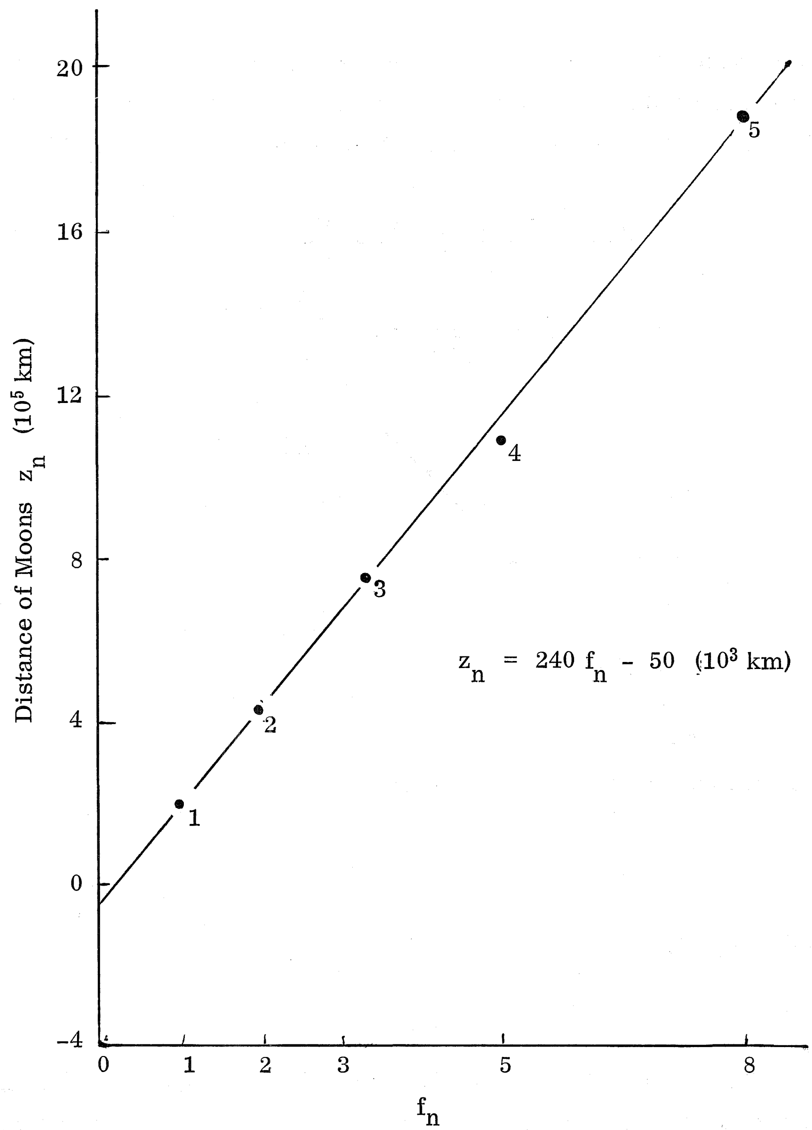

Fig. 1 First Moons of Jupiter

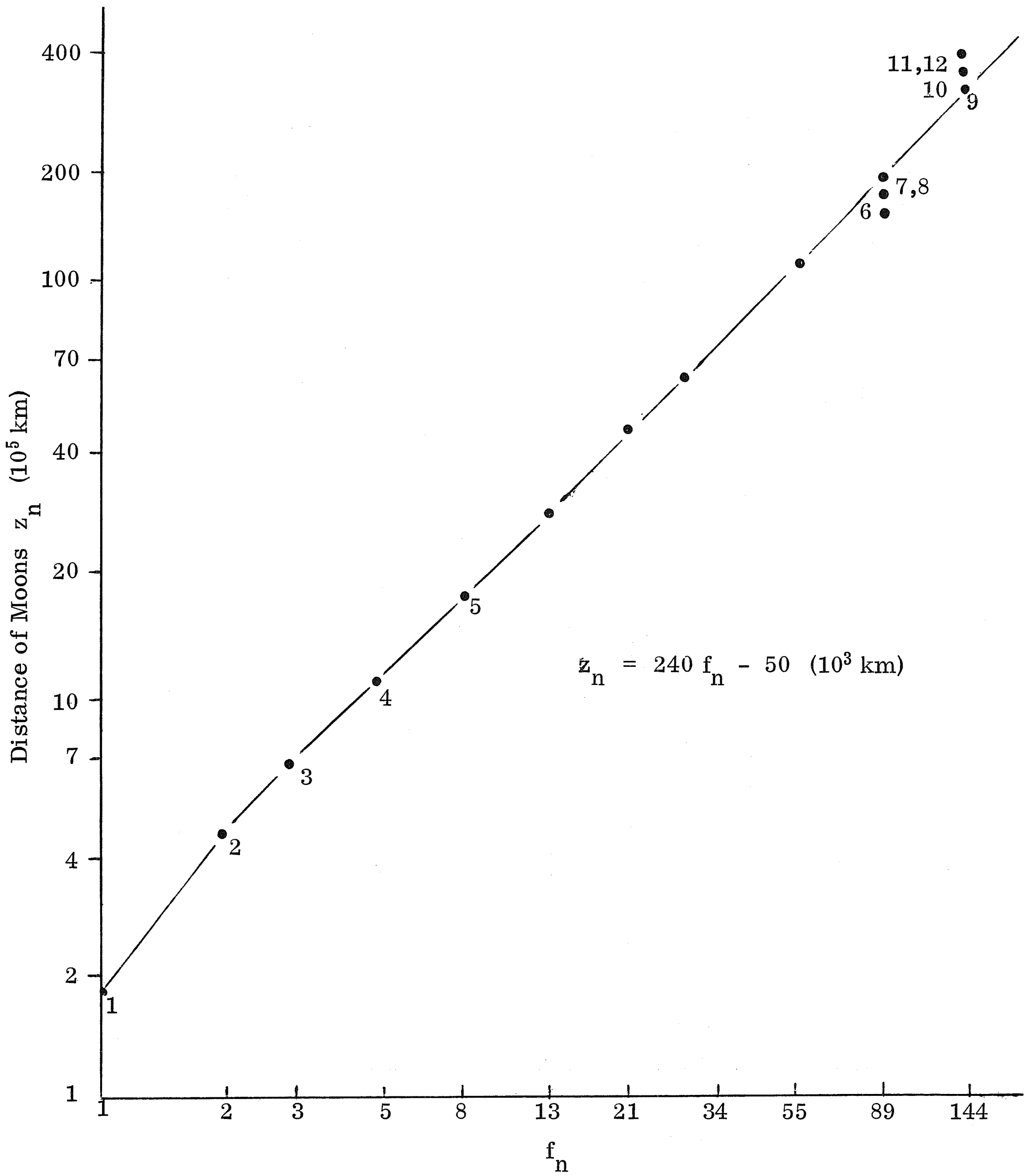

Fig. 2 Moons of Jupiter

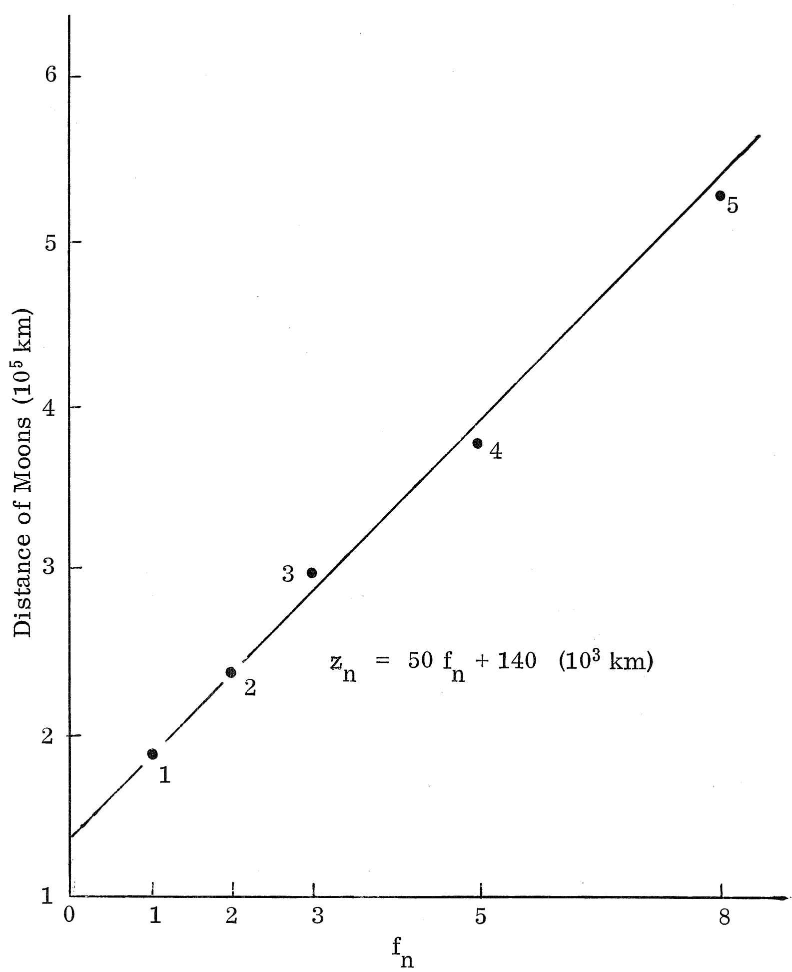

Fig. 3 First Moons of Saturn

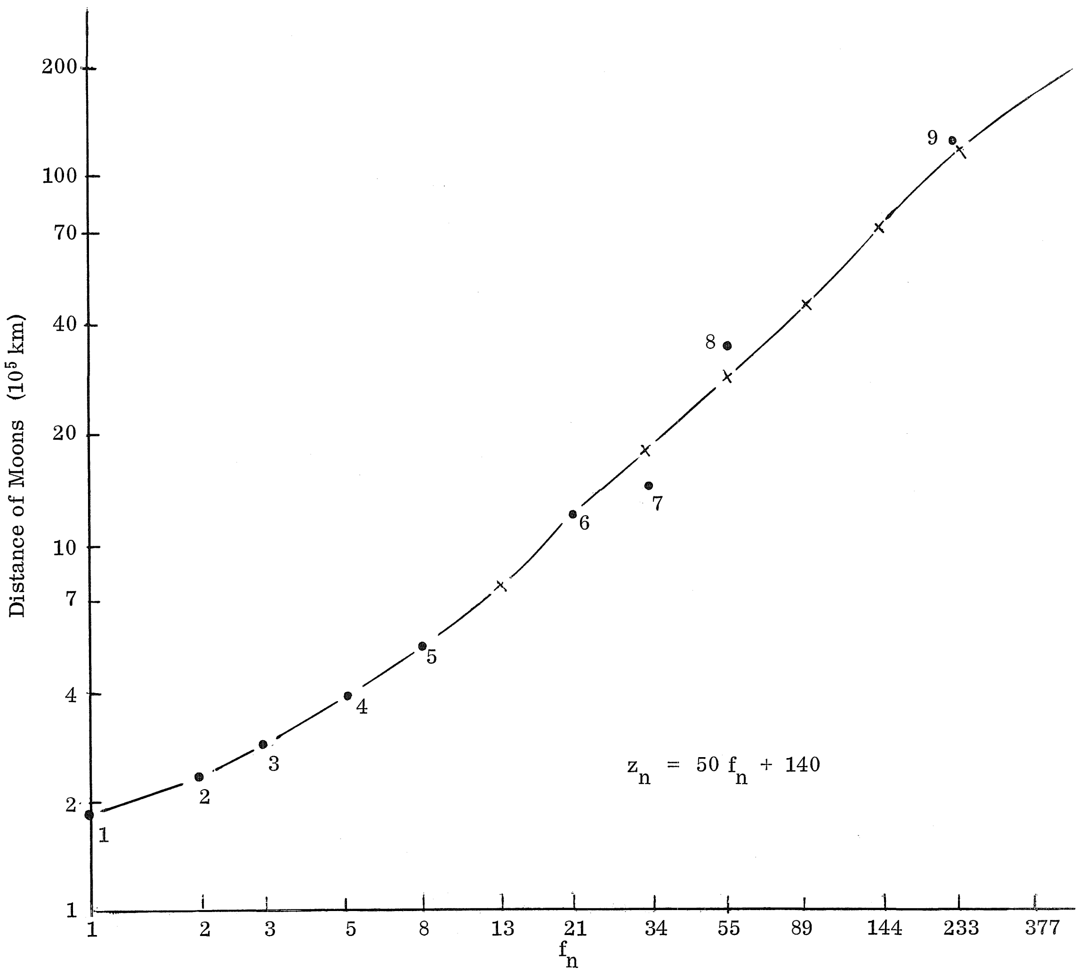

Fig. 4 Moons of Saturn

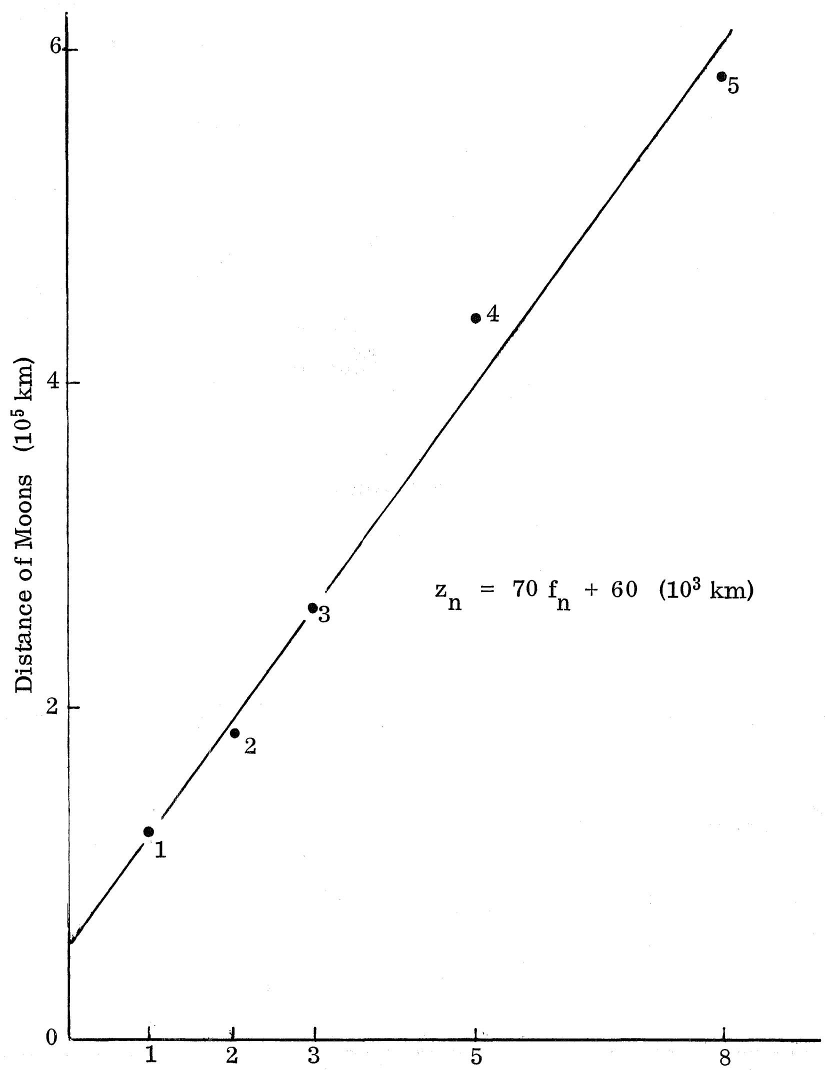

Fig. 5 Moons of Uranus

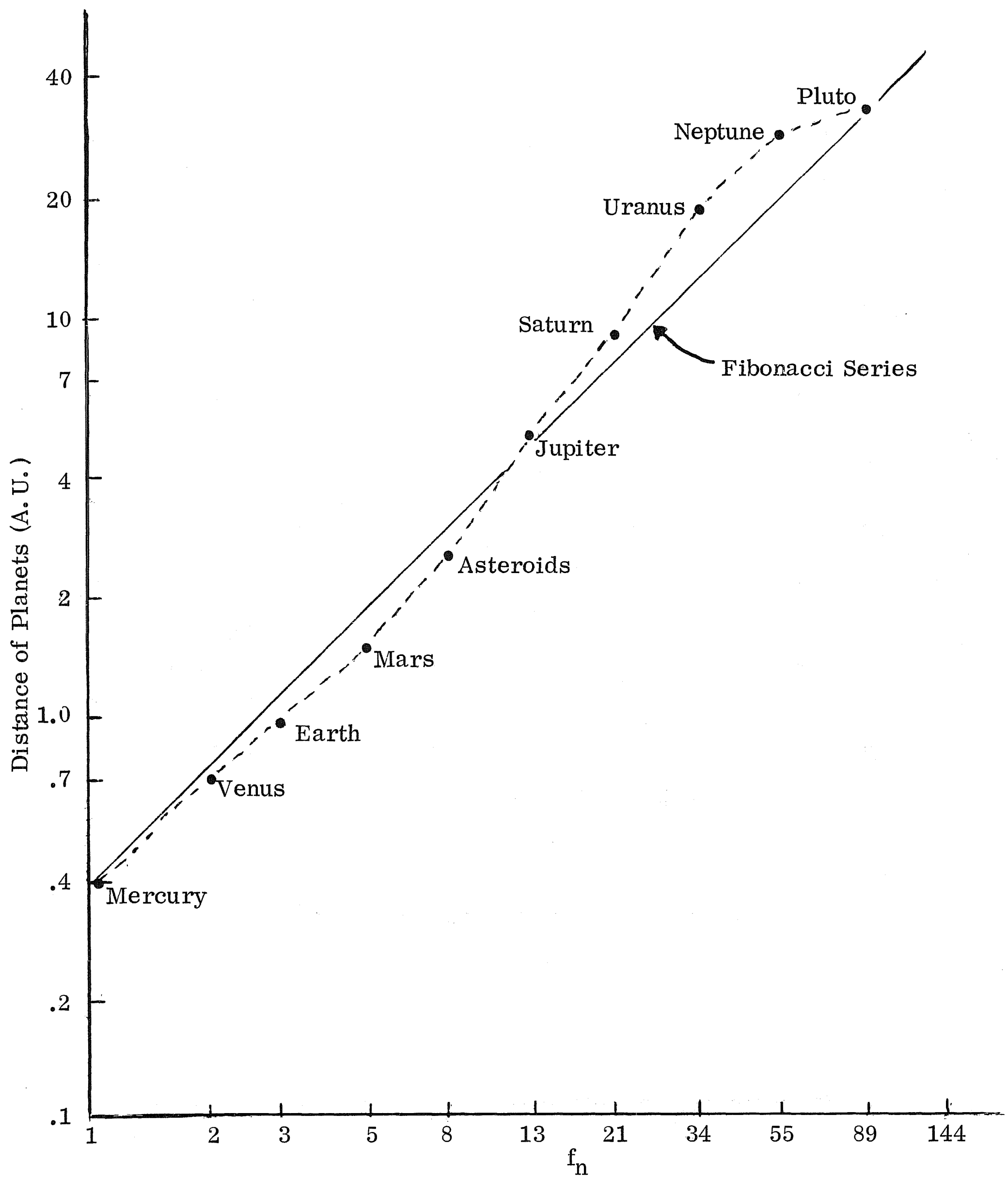

Fig. 6 The Planets

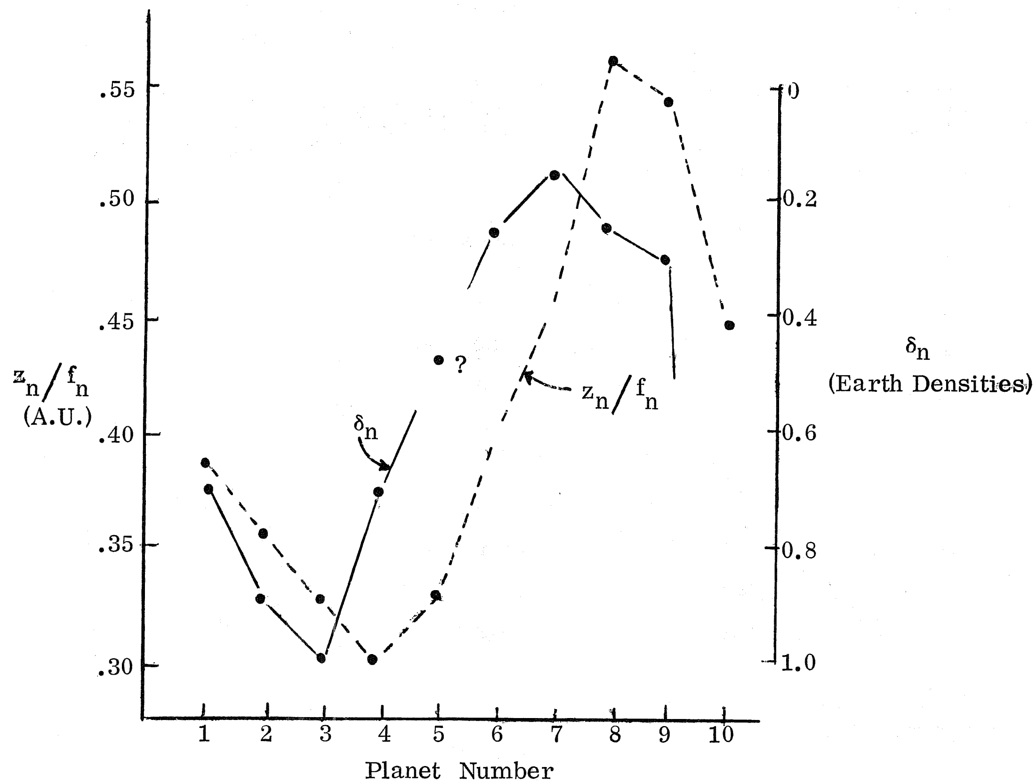

Fig. 7 z_n/f_n and δ_n for Planet Number n each planet. Superimposed upon this figure are the respective planet densities δ_n (in terms of earth densities). Figure 8 is a plot of z_n/f_n against δ_{n-1} which yields

$$z_n/f_n = 0.56 - 0.28\,\delta_{n-1}$$

(in astronomical units). The actual form of the density dependence of the offset is not readily determinable. It is conjectured from further study that the offset of a particular planet is either due to the average density of that planet and the planet previous, or is due to some weighting of previous densities with terms of the Fibonacci Series.

4. CONCLUSIONS

The significance of the values of m and b for the moons of the planets is left for subsequent interpretation. The quantity m may be regarded as a planetary scale length although neither m, b, or the ratio b/m seems to bear any relationship to such planetary parameters as density, mass, or volume.

The fact that the distance of the moons follows the Fibonacci Series means that a particular moon's position is dependent upon the positions of the previous two moons closer to the primary. Also, the moons seem to reside and, in the case of Jupiter, even congregate at "potential levels" predicted by the series.

5. ADDENDUM

The offset of a planet from the Fibonacci Series f_n being a function of the density of the previous planet is found to hold also for the offset of the moons of Jupiter and Saturn. The "normalized" offset for the moons of these two planets and for the planets of the sun is of the form,

$$\frac{z_n + C}{\overline{m}\,f_n} = 1.0 - 0.09\,\delta_{n-1} \quad \text{(cgs)}$$

although the reliability of the density data is poor.

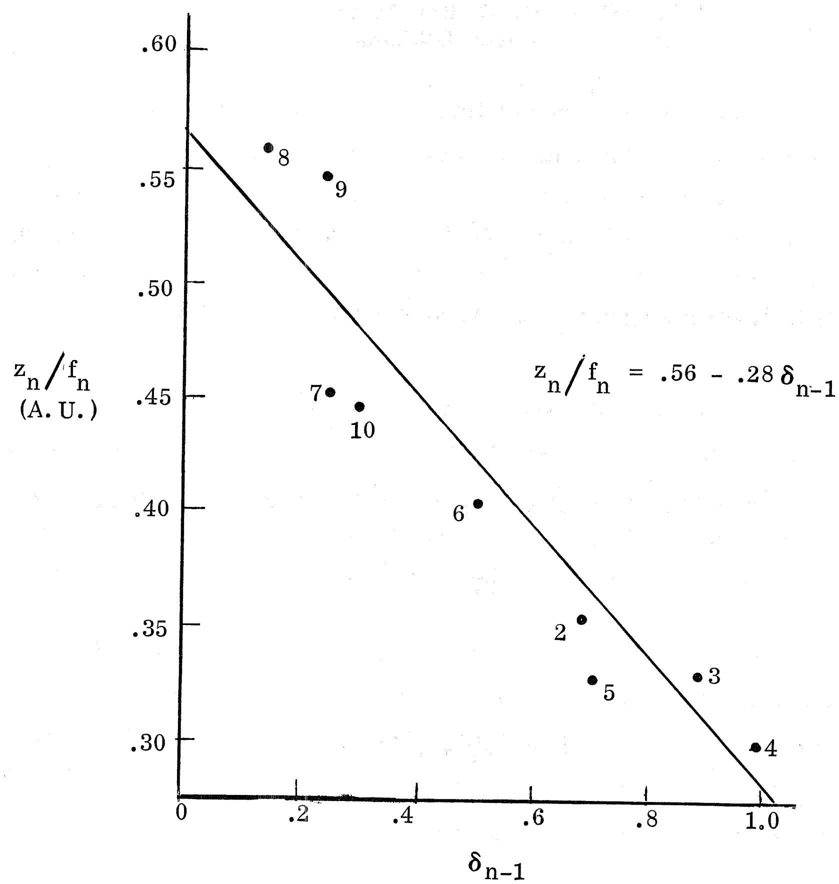

Fig. 8 "Offset" versus Previous Planet Density

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