FIBONACCI AND LUCAS NUMBERS TEND TO OBEY BENFORD'S LAW

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In numbers that appear in tables of physical and chemical constants. and similar tabulations, the digit 1 appears as first digit almost three times more often, as one would expect.

This phenomenal distribution of first digit is known at least for over 30 years when the physicist Frank Benford published a paper on this subject [1].

In his paper, Benford offered a general "law of anomalous numbers. " The probability that a random decimal begins with digit p is
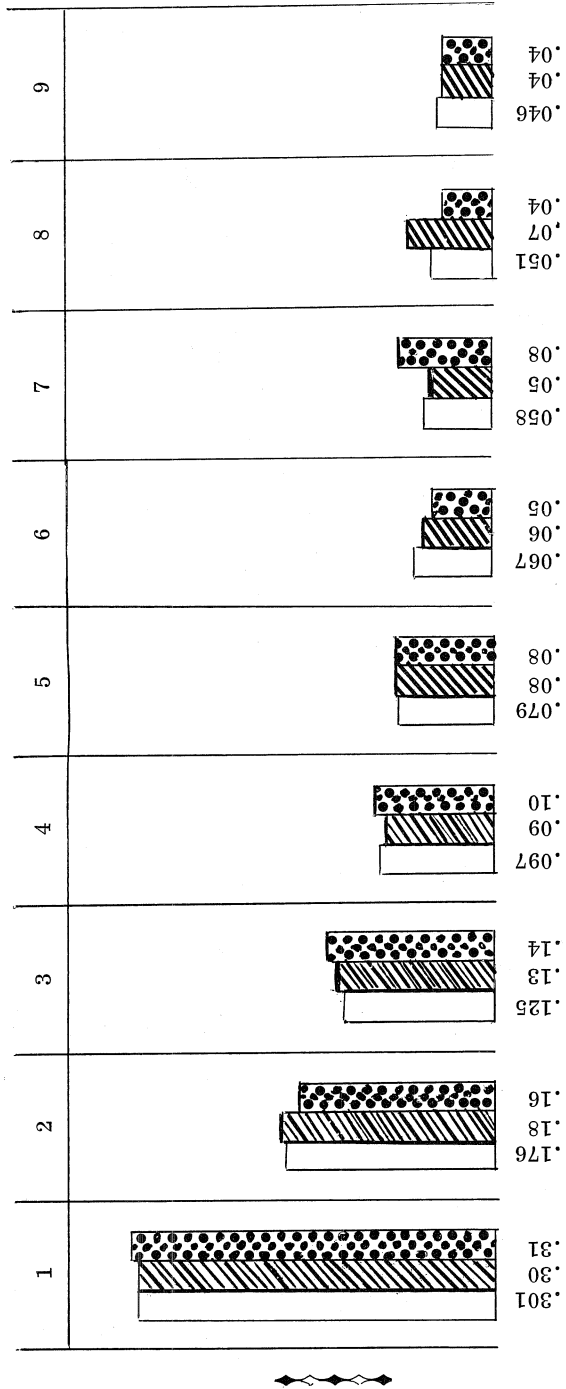
$$\log (p + 1) - \log p \ ,$$

where logarithms are based on 10.

Now the mathematician Ralph A. Raimi has, in [2], recently concerned himself with analysis of this number phenomenon, but did not refer to Fibonacci and Lucas numbers.

It seems that the first digits of Fibonacci and Lucas numbers tend to obey very closely the formula of probability offered by Benford. The chart on the following page shows the relation of frequencies for the first 100 Fibonacci Numbers and the first 100 Lucas numbers with frequencies of Benford's law. It would be interesting to make use of much more than 100 Fibonacci and Lucas numbers for the purpose of further analyzing Benford's law.

REFERENCES

1. Frank Benford, Proceedings of the American Philosophical Society, Vol. 78, No. 4, pp. 551-572 (March 31, 1938).
2. Ralph A. Raimi, Scientific American, Vol. 221, No. 6, pp. 109-120 (December 1969).

9
.046
.04
.04

8
.051
.07
.04

7
.058
.05
.08

6
.067
.06
.05

5
.079
.08
.08

4
.097
.09
.10

3
.125
.13
.14

2
.176
.18
.16

1
.301
.30
.31

The frequencies according to Benford's law are given by ☐ bars and numbers under them. The frequencies of first digits of the first 100 Fibonacci and Lucas numbers are given by ▨ and ▦ bars, respectively, and the numbers under them.

RELATION OF FREQUENCIES FOR FIBONACCI AND LUCAS NUMBERS WITH FREQUENCIES OF BENFORD'S LAW