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$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

## 4. EXTENSION TO OTHER NUMERICAL TRIANGLES

Although we have established the equality of products over the selected $m$ sets of $(m + 1)$ elements where the elements are multinomial coefficients, the results remain valid when the sequence $1, 2, 3, \cdots, n, \cdots$ in the multinomial coefficients is replaced by the Fibonacci Sequence $F_1, F_2, F_3, \cdots, F_n, \cdots$ ($F_1 = 1$, $F_2 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for $n \geq 1$.) The generalized binomial coefficients in [3] are integers and the generalized multinomial coefficients are integers in [2]. This is enough to guarantee the validity of both the theorems. This occurs because the identification of the various factors was independent of the particular function. In the case of the original theorem, $f(n) = n!$. In the case of the extension, $f(n) = F_1F_2 \cdots F_n$. Thus

$$\binom{k_1 + k_2 + \cdots + k_m}{k_1, k_2, k_3, \cdots, k_m} = \frac{f(k_1 + \cdots + k_m)}{f(k_1)f(k_2) \cdots f(k_m)} = \frac{\prod_{i=1}^{k_1+\cdots+k_m} F_i}{\prod_{i=1}^{k_1} F_i \prod_{i=1}^{k_2} F_i \cdots \prod_{i=m}^{k_m} F_i}$$

(as in [2]), instead of

$$\binom{k_1 + \cdots + k_m}{k_1, k_2, \cdots, k_m} = \frac{(k_1 + k_2 + \cdots + k_m)!}{k_1!k_2! \cdots k_m!} .$$

The corresponding $N$ is

$$N = \frac{\prod_{i=1}^{n-1} F_i \left(\prod_{i=1}^{n} F_i\right)^{m-1} \prod_{i=1}^{n+1} F_i}{\prod_{i=1}^{m}\left(\prod_{j=1}^{k_i-1} F_j \left(\prod_{j=1}^{k_i} F_j\right)^{m-1} \prod_{j=1}^{k_i+1} F_j\right)},$$

where $n = k_1 + k_2 + \cdots + k_m$.

## REFERENCES


1. Walter Hansell and V. E. Hoggatt, Jr., "The Hidden Hexagon Squares," Fibonacci Quarterly, Vol. 9, No. 2, p. 120.
2. Eugene Kohlbecker, "On a Generalization of Multinomial Coefficients for the Fibonacci Sequence," Fibonacci Quarterly, Vol. 4, No. 1, pp. 307-312.
3. V. E. Hoggatt, Jr., "Generalized Binomial Coefficients and the Fibonacci Numbers," Fibonacci Quarterly, Vol. 5, No. 4, pp. 383-400.
4. Henry W. Gould, "Equal Products of Generalized Binomial Coefficients," Fibonacci Quarterly, Vol. 9, No. 4, pp. 337-346.


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where

$$H_{n+2} = H_{n+1} + H_n .$$

The following identities were obtained from (13.2):

$$(13.10) \qquad H_{4n+4+p} - H_{2n+2+p} = \sum_{i=0}^{n} \binom{2n+1-i}{i} H_{3i+3+p},$$

$$H_{8n+8+p} - 5^{n+1} H_{4n+4+p}$$

$$(13.11) \qquad = 3 \sum_{i=0}^{[n/2]} \binom{2n+1-2i}{2i} 5^i H_{12i+3+p}$$

$$+ 3 \sum_{j=0}^{[(n-1)/2]} \binom{2n-2j}{2j+1} 5^j (H_0 L_{12j+8+p} + H_1 L_{12j+9+p}) .$$