$$\mathbf{N} = \frac{\prod\limits_{i=1}^{n-1} \mathbf{F}_i \left(\prod\limits_{i=1}^{n} \mathbf{F}_i\right)^{m-1} \prod\limits_{i=1}^{n+1} \mathbf{F}_i}{\prod\limits_{i=1}^{m} \left(\prod\limits_{j=1}^{k_i-1} \mathbf{F}_j \left(\prod\limits_{j=1}^{k_i} \mathbf{F}_j\right)^{m-1} \prod\limits_{j=1}^{k_i+1} \mathbf{F}_j\right)} \ ,$$

where $n = k_1 + k_2 + \cdots + k_m$.

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where

$$H_{n+2} = H_{n+1} + H_n$$
.

The following identities were obtained from (13.2):

(13.10)
$$H_{4n+4+p} - H_{2n+2+p} = \sum_{i=0}^{n} \binom{2n+1-i}{i} H_{3i+3+p} ,$$

$$\begin{array}{ll} H_{8n+8+p} - 5^{n+1} H_{4n+4+p} \\ \\ (13.11) & = 3 \sum_{i=0}^{\left[n/2\right]} \binom{2n+1-2i}{2i} 5^{i} H_{12i+3+p} \\ \\ & + 3 \sum_{j=0}^{\left[(n-1)/2\right]} \binom{2n-2j}{2j+1} 5^{j} \left(H_{0} L_{12j+8+p} + H_{1} L_{12j+9+p}\right) \end{array}.$$

Many more Fibonacci identities are readily obtainable from (13.1) and (13.2).

14. REMARKS ON THE PAPER BY HOGGATT, PHILLIPS, AND LEONARD [5]

All the 22 identities in [5] are special cases of our general results. The 22 identities appear in the Master's thesis of Leonard [6]. The notation (A, 1.6) means that identity A of [5] is a special case of our identity (1.6). Thus, we have the remaining identity pairings for special cases of our results: (B, 1.8), (C, 4.7), (D, 4.3), (E, 1.6), (F, 1.8), (G, 4.7), (H, 4.3), (I, 1.15), (J, 1.16), (K, 1.11), (L, 1.13), (M, 1.9), (N, 1.12), (P, 4.8), (Q, 4.4), (R, 4.5), (S, 4.9), (T, 1.16), (U, 1.15), (V, 1.16), and (W, 1.15).

Since A and E are obtained as special cases of our (1.6), A and E are therefore not independent, i.e., by a change of parameters, A can be transformed to E and vice versa. Thus, a perusal of the above pairings gives us the following dependent identity grouping: (A,E; 1.6), (B,F; 1.8), (I,U,W; 1.15), (J,T,V; 1.16), (D,H; 4.3), (C,G; 4.7). Since K, L. M,N, P, Q, R, and S are independent, the 22 identities A, B, ···, W, contains now only 14 independent identities.

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