# Almost Quantum Correlations are Inconsistent with Specker's Principle — Extended Abstract

Tomáš Gonda[1 2], Ravi Kunjwal[1], David Schmid[1 2], Elie Wolfe[1], and Ana Belén Sainz[1]

[1]Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada, N2L 2Y5

[2]Dept. of Physics and Astronomy, University of Waterloo, Waterloo, Ontario, Canada, N2L 3G1

March 28, 2018

Preprint of the full article can be found at arxiv.org/pdf/1712.01225.pdf.

The advent of quantum theory was accompanied by many conceptual controversies over the failure of intuitions from classical physics, e.g. the existence of wave-particle duality, the fundamental indeterminism apparent from the Born rule, and the nonseparability epitomized by entanglement, as pointed out by Einstein, Podolsky and Rosen [1, 2]. More recently, we have witnessed the emergence of quantum information theory and a surge of interest in quantum foundations. As a consequence, there has been remarkable progress in proving theorems concerning ways in which Nature fails to be classical, given a well-defined notion of classicality that mathematically formalizes some intuition from classical physics.

Bell nonlocality [3] and (Kochen-Specker) contextuality [4] are phenomena that cannot be explained by any classical model of the world, where the respective notions of classicality are called *local causality* and *Kochen-Specker (KS) noncontextuality*. Although these notions of classicality are mathematically similar, they are conceptually very distinct. Both notions posit the possibility of joint assignment of outcomes to all measurements in a given experiment, and as a result the existence of a joint probability distribution over these outcomes. However, local causality does not appeal to the structure of measurements in the theory, while KS-noncontextuality does, by referring to "sharp measurements" only[1] in the operational theory describing the experiment. The latter, then, fails to make sense as a notion of classicality for unsharp measurements. The operational notion of noncontextuality due to Spekkens [5] resolves this problem and provides an overarching framework that is much more generally applicable [6], and reduces to KS-noncontextuality in the case of projective measurements in quantum theory.

Given the fact that physical theories are always amenable to revision in the light of new evidence, one might question whether Nature is strictly quantum. That is, are there deviations from quantum theory in Nature, and if so, where should one look for them [7]? There has been much effort recently to identify the physical properties that single out our quantum world from the space of hypothetical alternatives. By doing this, not only can we learn more about quantum theory itself, but we also gain insight about where one might or might not hope to find failures of quantum theory.

In the past two decades, there have been two major lines of research tackling this problem. On the one hand, there have been derivations of the *structure* of quantum theory from a set of simple axioms [8–18]. On the other hand, there is the program of characterizing only the *statistical* aspect of quantum

Tomáš Gonda: tgonda@perimeterinstitute.ca

[1]In quantum theory, the notion of sharp measurements corresponds to projective measurements.

theory, i.e. recovering quantum correlations. This second approach has made considerable progress in articulating principles that are satisfied by quantum correlations, but a key question remains: Are there any principles that *uniquely* identify the set of quantum correlations in the space of all conceivable correlations?

The study of principles to single out the set of quantum correlations led to the conception of almost quantum correlations [19]. Initially defined only within Bell scenarios [19], almost quantum correlations have also been subsequently defined within general (KS-)contextuality scenarios [20]. This set of correlations strictly contains the quantum set, yet has the following remarkable property: almost quantum correlations satisfy all principles proposed so far to characterize quantum correlations[2] [19]. In particular, almost quantum correlations satisfy the principle of consistent exclusivity (CE), as is depicted in figure 1. Therefore, it is desirable to identify a principle capable of discriminating quantum from almost quantum correlations.

Lately, the focus has slightly shifted instead towards characterizing what kinds of physical theories could lead to almost quantum statistics. The hope is to clarify the relevant structural differences between such theories and the quantum one. For example, recent results show that any general probabilistic theory [21] giving rise to almost quantum correlations would have to violate the no-restriction hypothesis [22, 23].

This is where the so-called Specker's principle enters the story. Ernst Specker considered a particular feature of quantum theory to be especially fundamental, namely that pairwise joint measurability implies global joint measurability for sharp measurements [vimeo.com/52923835 (2009)]. To date, it seemed that Specker's principle failed to single out quantum theory from the space of all general probabilistic theories. In particular, consistent exclusivity — an important consequence of Specker's principle — is satisfied by both quantum and almost quantum correlations.

However, in the article we identify several equivalent formulations of Specker's principle that motivate distinct consequences for the allowed sets of correlations. Figure 1 summarizes the one that are of main interest to us. It also depicts the known implications that hold among these statements for any given theory and the set of correlations generated by some subset of its measurements (called sharp measurements). Through numerical analysis, we have been able to show that one of these consequences — pairwise sufficiency for probabilistic models — is in fact violated by almost quantum correlations. This, in turn, implies the main result saying that "there is no set of measurements, in any general probabilistic theory, which both gives rise to almost quantum correlations and satisfies Specker's principle".

As a corollary we obtain a way to discern quantum and almost quantum correlations through the above-mentioned property of pairwise sufficiency, an explicit definition of which can be found in the main text. Alternatively, we conclude that Specker's principle is capable of distinguishing quantum theory from almost quantum theory.

In figure 1, one can notice a property which is neither green nor red — all-but-one sufficiency for probabilistic models. This means that it is unknown whether it holds for almost quantum correlations or not. We prove that almost quantum correlations do satisfy this property if one restricts to measurements with two outcomes only. The general case is left open though. It would be interesting if it turns out that all-but-one sufficiency for probabilistic models is indeed satisfied by almost quantum correlations, given the close connection of the pairwise sufficiency and all-but-one sufficiency properties. One could then hope to give almost quantum (or related) correlations a more operational meaning.

[2] Strictly speaking, almost quantum correlations have not been proven to satisfy the principle of Information Causality yet; only numerical evidence exists for that claim.

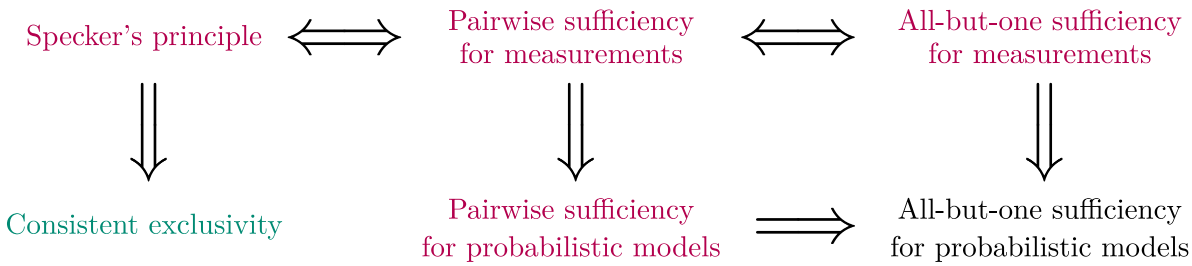


**Figure 1:** Some of the consequences of Specker's principle and the implications known to hold among them. The top row contains principles pertaining to the structure of measurements, while the bottom row contains principles pertaining to sets of probabilistic models. Text colour depicts whether a given statement holds in an almost quantum theory: the green statement is satisfied, the red statements are violated, and the black statement is the subject of conjecture in the main article.

By studying contextuality scenarios on a single system, we have thus found a fundamental difference between quantum theory and any potential almost quantum theory, complementing the results by Sainz et al. [23]. There, a different no-go theorem (pertaining to almost quantum theories and the no-restriction hypothesis) was inferred based on considerations of Bell scenarios involving multiple subsystems.

Remarkably, our results imply that in any general probabilistic theory, the structure of measurements reproducing almost quantum models is in contradiction with Specker's principle. Accordingly, the notion of sharpness proposed in references [24, 25] cannot be used to recover the almost quantum correlations. This result runs counter to sentiments implicit in earlier literature. Previously, almost quantum models appeared to be the purest embodiment of Specker's principle [19, 26–30], in the sense that they are uniquely identified[3] by consistent exclusivity and closure under wirings [20, Thrm. 7.6.2].

For advocates of Specker's principle, our results challenge the possible physical significance of the almost quantum correlations [31]. More importantly, however, our findings restore the prominence of Specker's principle as a means for identifying the essence of quantum theory. The violation of Specker's principle by almost quantum models demonstrates that holistic considerations of Specker's principle enables greater insight than consistent exclusivity alone.

## References

[1] G. Bacciagaluppi and A. Valentini, *Quantum theory at the crossroads: reconsidering the 1927 Solvay conference* (Cambridge University Press, 2009).

[2] A. Einstein, B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" Phys. Rev. **47**, 777 (1935).

[3] J. S. Bell, "On the Einstein-Podolsky-Rosen paradox," Physics **1**, 195 (1964).

[4] S. Kochen and E. P. Specker, "The problem of hidden variables in quantum mechanics," J. Math. Mech. **17**, 59 (1967).

[5] R. W. Spekkens, "Contextuality for preparations, transformations, and unsharp measurements," Phys. Rev. A **71**, 052108 (2005).

[3] Almost quantum correlations are uniquely identified under the assumptions that the set of correlations allowed in Nature contains the quantum one, and by considering the ramifications of consistent exclusivity even in the limit of many independent copies of the scenario.

[6] R. Kunjwal, "Beyond the Cabello-Severini-Winter framework: making sense of contextuality without sharpness of measurements," arXiv:1709.01098 (2017).

[7] M. D. Mazurek, M. F. Pusey, K. J. Resch, and R. W. Spekkens, "Experimentally bounding deviations from quantum theory in the landscape of generalized probabilistic theories," arXiv:1710.05948 (2017).

[8] L. Hardy, "Quantum Theory From Five Reasonable Axioms," arXiv:quant-ph/0101012 (2001).

[9] B. Dakic and C. Brukner, "Quantum theory and beyond: is entanglement special?" arXiv preprint arXiv:0911.0695 (2009).

[10] L. Masanes and M. P. Müller, "A derivation of quantum theory from physical requirements," New Journal of Physics **13**, 063001 (2011).

[11] L. Masanes, M. P. Müller, R. Augusiak, and D. Pérez-García, "Existence of an information unit as a postulate of quantum theory," Proceedings of the National Academy of Sciences **110**, 16373 (2013), http://www.pnas.org/content/110/41/16373.full.pdf .

[12] M. P. Müller and L. Masanes, "Information-theoretic postulates for quantum theory," in *Quantum Theory: Informational Foundations and Foils*, edited by G. Chiribella and R. W. Spekkens (Springer Netherlands, Dordrecht, 2016) pp. 139–170.

[13] G. Chiribella, G. M. D'Ariano, and P. Perinotti, "Quantum from principles," in *Quantum Theory: Informational Foundations and Foils*, edited by G. Chiribella and R. W. Spekkens (Springer Netherlands, Dordrecht, 2016) pp. 171–221.

[14] L. Hardy, "Reconstructing quantum theory," in *Quantum Theory: Informational Foundations and Foils*, edited by G. Chiribella and R. W. Spekkens (Springer Netherlands, Dordrecht, 2016) pp. 223–248.

[15] B. Dakić and Č. Brukner, "The classical limit of a physical theory and the dimensionality of space," in *Quantum Theory: Informational Foundations and Foils*, edited by G. Chiribella and R. W. Spekkens (Springer Netherlands, Dordrecht, 2016) pp. 249–282.

[16] R. Oeckl, "A local and operational framework for the foundations of physics," arXiv:1610.09052 (2016).

[17] P. A. Höhn, "Toolbox for reconstructing quantum theory from rules on information acquisition," arXiv preprint arXiv:1412.8323 (2014).

[18] P. A. Höhn and C. S. P. Wever, "Quantum theory from questions," Phys. Rev. A **95**, 012102 (2017).

[19] M. Navascués, Y. Guryanova, M. J. Hoban, and A. Acín, "Almost quantum correlations," Nat. Comm. **6**, 6288 (2015).

[20] A. Acín, T. Fritz, A. Leverrier, and A. B. Sainz, "A Combinatorial Approach to Nonlocality and Contextuality," Comm. Math. Phys. **334**, 533 (2015).

[21] J. Barrett, "Information processing in generalized probabilistic theories," Phys. Rev. A **75**, 032304 (2007).

[22] G. Chiribella, G. M. D'Ariano, and P. Perinotti, "Probabilistic theories with purification," Phys. Rev. A **81**, 062348 (2010).

[23] A. B. Sainz, Y. Guryanova, A. Acín, and M. Navascués, "Almost quantum correlations violate the no-restriction hypothesis," arXiv:1707.02620 (2017).

[24] G. Chiribella and X. Yuan, "Measurement sharpness cuts nonlocality and contextuality in every physical theory," arXiv:1404.3348 (2014).

[25] G. Chiribella and X. Yuan, "Bridging the gap between general probabilistic theories and the device-independent framework for nonlocality and contextuality," Info. & Comp. **250**, 15 (2016).

[26] A. Cabello, "Specker's fundamental principle of quantum mechanics," arXiv:1212.1756 (2012).

[27] A. Cabello, S. Severini, and A. Winter, "(Non-)Contextuality of Physical Theories as an Axiom," arXiv:1010.2163 (2010).

[28] B. Yan, "Quantum correlations are tightly bound by the exclusivity principle," Phys. Rev. Lett. **110**, 260406 (2013).
[29] A. Cabello, "Exclusivity principle and the quantum bound of the bell inequality," Phys. Rev. A **90**, 062125 (2014).
[30] B. Amaral, M. T. Cunha, and A. Cabello, "Exclusivity principle forbids sets of correlations larger than the quantum set," Phys. Rev. A **89**, 030101 (2014).
[31] F. Dowker, J. Henson, and P. Wallden, "A histories perspective on characterising quantum non-locality," New Journal of Physics **16**, 033033 (2014).