Entanglement is necessary for emergent classicality in all physical theories

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Abstract

One of the most striking features of quantum theory is the existence of entangled states, responsible for Einstein's so called "spooky action at a distance". These states emerge from the mathematical formalism of quantum theory, but to date we do not have a clear idea of the physical principles that give rise to entanglement. Why does nature have entangled states? Would any theory superseding classical theory have entangled states, or is quantum theory special? One important feature of quantum theory is that it has a classical limit, recovering classical theory through the process of decoherence. We show that any theory with a classical limit must contain entangled states, thus establishing entanglement as an inevitable feature of any theory superseding classical theory.

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1 Introduction

Entanglement and non-locality are two of the features of quantum theory that clash most strongly with our classical preconceptions as to how the universe works. In particular, they create a tension with the other major theory of the twentieth century: relativity. This is most clearly illustrated by Bell's theorem [2, 16], in which certain entangled states are shown to violate local realism by allowing for correlations that cannot be explained by classical causal structures [16]. In this paper we ask whether entanglement is a surprising feature of nature, or whether it should be expected in any non-classical theory? Could a scientist with no knowledge of quantum theory have predicted the existence of entangled states based solely on the premise that their classical understanding of the world was incomplete?

Any such scientist could reasonably postulate the existence of a classical regime - in that, whatever theory describes reality must be able to behave like classical theory in some limit. Although this is a very natural assumption, given that we frequently observe systems behaving classically, we show that it imparts very strong constraints on the structure of any non-classical theory. Thus to answer these questions, we explore all theories that have a classical limit [17, 7].

In quantum theory this is formalised by *decoherence maps*, which take quantum systems to classical states with respect to some basis. Physically, decoherence maps represents a quantum system interacting with some inaccessible environment resulting in the loss of quantum coherences. Inspired by this we develop a generalization of decoherence maps for arbitrary operationally defined theories (see [14, 8, 5] for a related process-theoretic approach). We consider all theories that can decohere to classical theory and show that any such theory either contains entangled states, or is simply classical theory itself. Thus, the existence of these classically counter-intuitive entangled states present in quantum theory can be understood as arising from, and being necessary for, the existence of a classical world. This result hints towards the possibility that other counter-intuitive features of quantum theory could be derived from its accommodation of a classical limit, and paves the way for deriving the features of post-classical and post-quantum theories from the existence of this limit.

2 Setup

To begin to pose questions about how different physical features of theories relate we makes use of the generalised probabilistic theories (GPT) framework [6, 1], which is broad enough to describe any theory of nature admitting an operational description. For a full introduction to this framework see [1, 4]. Here we will simply provide definitions for the key concepts used in this work, the most important of which is entanglement.

Definition 1 (Entanglement). A state ψ belonging to the bipartite state space $\mathcal{S}_1 \otimes \mathcal{S}_2$ is entangled iff it cannot be written in the following form

$$\psi = \sum_i p_i s_i \otimes s'_i \quad \sum_i p_i = 1, \;\; p_i \geq 0 \;\; \forall\, i$$

where $s_i \in \Omega_{\mathcal{S}_1}$, $s'_i \in \Omega_{\mathcal{S}_2}$, i.e. a state is entangled if it cannot be seen as the convex combination of product states. Note that here $\otimes$ simply denotes parallel composition of states and should not be interpreted as the vector space tensor product.

In the paper we show that such entangled states are a feature of any non-classical theory which can decohere to classical theory. As mentioned earlier, it is physically well motivated to postulate that in some regime systems must be able to behave classically. But, moreover, the GPT framework itself is fundamentally built on the assumption that we have a *classical interface* with the world. We can choose, potentially using classical randomness, which experiment to perform, and we can characterise states, effects and transformations in terms of classical probability distributions that we obtain from experiments. However, ultimately this classical interface should be explainable from the theory itself rather than just being an external structure. This is indeed the case in quantum theory, where we can view the classical interface as an effective description of *decohered* quantum systems. It therefore seems like any well-founded GPT should have an analogous decoherence mechanism so as to explain how it gives rise to the classical interface.

Definition 2 (Decoherence maps). A decoherence map for a system $\mathcal{S}$ is a map $D_{\mathcal{S}} : \mathcal{S} \to \mathcal{S}$ satisfying the following conditions:

1. Physicality: the decoherence map is a physical map, typically considered to be arising from an interaction with some environmental system that is then discarded, and hence must satisfy all of the constraints on transformations in a GPT. In particular, it must be *linear* and map states to states.

2. Idempotence: in quantum theory the decoherence map destroys the coherences between the basis states, and so applying it a second time does nothing to the state. In general, the decoherence map should restrict the state space to a classical subspace which is invariant under repeat applications.

3. Purity-decreasing: the decoherence map arises from losing information to an environment and as such it cannot increase our knowledge of the input state. Therefore $D[\rho]$ cannot be strictly purer than ρ for any input state ρ. For example in quantum theory a decoherence map will not map mixed states to pure states.

4. Classical states: the image of the decoherence map is a classical state space: $D_{\mathcal{S}}(\Omega_{\mathcal{S}}) = \Delta_{N(\mathcal{S})}$. However, we don't just want to reproduce the states of classical theory, but the full theory including its dynamical and probabilistic structure.

5. Classical effects: for every effect in classical theory there is some effect in the full theory that behaves as the classical effect when we restrict to $\Delta_{N(\mathcal{S})}$. This can be formalised as for all $e_{\text{classical}}$ there exists $e \in \mathcal{E}_{\mathcal{S}}$ such that $e_{\text{classical}} = e \circ D_{\mathcal{S}}$.

6. Classical (reversible) dynamics: similarly for (reversible) transformations we expect for any classical (reversible) transformation t there is a corresponding post-classical (reversible) transformation with the same action on the image of $D_{\mathcal{S}}$. This can be formalised as for all classical reversible transformations $T_{\text{classical}}$ there exists some reversible $T \in \mathcal{T}_{\mathcal{S}}$ such that $T_{\text{classical}} = T \circ D_{\mathcal{S}}$.

7. Composites: finally, we expect decoherence to interact suitably with composition, i.e. if system $\mathcal{S}_1$ decoheres to Δ_N via $D_{\mathcal{S}_1}$ and system $\mathcal{S}_2$ to Δ_M via $D_{\mathcal{S}_2}$, then the composite system $\mathcal{S}_1 \otimes \mathcal{S}_2$ can decohere to $\Delta_N \otimes \Delta_M = \Delta_{NM}$ via $D_{\mathcal{S}_1} \otimes D_{\mathcal{S}_2}$.

3 Results and discussion

Informally our result can be stated as:

Theories with non-trivial decoherence
must have entangled states.

The formal statement is given in [13, Result 3], but essentially what we prove is that: if we assume a) the theory to be non-classical (such that decoherence is non-trivial), and b) that the theory doesn't have any entangled states, then the only decoherence map possible is simply 'discard our system and prepare a fixed state'. The resulting classical system would be trivial, i.e. it only has a single state and hence if we want non-trivial decoherence we must have entangled states.

It therefore seems that entanglement, rather than being a surprising feature of nature, is an entirely inevitable feature of any post-classical theory. A natural question to ask is what other features of quantum theory can be reproduced simply by demanding that the theory has a classical limit? There are myriad other physical features that could be implied from the existence of a classical limit such as information causality [12], bit symmetry [10] and macroscopic locality [11] to name but a few.

Of particular interest would be deriving genuine device-independent non-locality. The existence of entangled states is in general a necessary but insufficient condition for observing violations of Bell inequalities. For example, non-separable states are present in the local theory of Spekken's toy model [15]. On the other hand, it has been shown that all entangled states in quantum theory display some hidden non-locality [3, 9]. By determining the additional structure present in quantum theory that gives this correspondence between entanglement and non-locality, it could be possible to derive the violation of Bell inequalities from purely physical postulates. Given the simplicity of the postulates used to derive the existence of entangled states, it is plausible that the postulates that give rise to Bell non-locality are similarly mundane.

This notion of decoherence has allowed us to define a new class of GPTs – those with a classical limit. There is clear physical motivation to consider this class. For example, if a theory were not to have such a limit then one would have to posit the existence of two fundamentally distinct types of systems, the classical systems (which are how we interact with the world) along with post-classical systems (which we cannot directly probe). Such a fundamental distinction appears unnatural, and so it seems that decoherence is a necessary feature of any sensible operational theory. However, whilst being a physically well-motivated class, it nonetheless provides a great deal of mathematical structure and as such gives a more powerful framework for studying generalised theories.

References

[1] Jonathan Barrett. Information processing in generalized probabilistic theories. *Physical Review A*, 75(3):032304, 2007.

[2] John S Bell. Physics 1, 195 (1964). *Rev. Mod. Phys*, 38:447, 1966.

[3] Francesco Buscemi. All entangled quantum states are nonlocal. *Physical review letters*, 108(20):200401, 2012.

[4] Giulio Chiribella, Giacomo Mauro D'Ariano, and Paolo Perinotti. Informational derivation of quantum theory. *Physical Review A*, 84(1):012311, 2011.

[5] Bob Coecke, John Selby, and Sean Tull. Two roads to classicality. In Bob Coecke and Aleks Kissinger, editors, Proceedings 14th International Conference on *Quantum Physics and Logic,* Nijmegen, The Netherlands, 3-7 July 2017, volume 266 of *Electronic Proceedings in Theoretical Computer Science*, pages 104–118. Open Publishing Association, 2018.

[6] Lucien Hardy. Quantum theory from five reasonable axioms. *arXiv preprint quant-ph/0101012*, 2001.

[7] Erich Joos, H Dieter Zeh, Claus Kiefer, Domenico JW Giulini, Joachim Kupsch, and Ion-Olimpiu Stamatescu. *Decoherence and the appearance of a classical world in quantum theory.* Springer Science & Business Media, 2013.

[8] Ciarán M Lee and John H Selby. A no-go theorem for theories that decohere to quantum mechanics. *arXiv preprint arXiv:1701.07449*, 2017.

[9] Lluís Masanes, Yeong-Cherng Liang, and Andrew C Doherty. All bipartite entangled states display some hidden nonlocality. *Physical review letters*, 100(9):090403, 2008.

[10] Markus P Müller and Cozmin Ududec. Structure of reversible computation determines the self-duality of quantum theory. *Physical review letters*, 108(13):130401, 2012.

[11] Miguel Navascués and Harald Wunderlich. A glance beyond the quantum model. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, page rspa20090453. The Royal Society, 2009.

[12] Marcin Pawłowski, Tomasz Paterek, Dagomir Kaszlikowski, Valerio Scarani, Andreas Winter, and Marek Żukowski. Information causality as a physical principle. *Nature*, 461(7267):1101–1104, 2009.

[13] Jonathan G Richens, John H Selby, and Sabri W Al-Safi. Entanglement is necessary for emergent classicality in all physical theories. *Physical review letters*, 119(8):080503, 2017.

[14] John Selby and Bob Coecke. Leaks: Quantum, classical, intermediate and more. *Entropy*, 19(4):174, 2017.

[15] Robert W Spekkens. Evidence for the epistemic view of quantum states: A toy theory. *Physical Review A*, 75(3):032110, 2007.

[16] Christopher J Wood and Robert W Spekkens. The lesson of causal discovery algorithms for quantum correlations: Causal explanations of bell-inequality violations require fine-tuning. *New Journal of Physics*, 17(3):033002, 2015.

[17] Wojciech Hubert Zurek. Quantum darwinism. *Nature Physics*, 5(3):181–188, 2009.