A channel-based framework for steering, non-locality and beyond

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Non-locality and steering are both non-classical phenomena witnessed in Nature as a result of quantum entanglement. It is now well-established that one can study non-locality independently of the formalism of quantum mechanics, in the so-called device-independent framework. With regards to steering, although one cannot study it completely independently of the quantum formalism, "post-quantum steering" has been described, that is steering which cannot be reproduced by measurements on entangled states but do not lead to superluminal signalling. In this work we present a framework based on the study of quantum channels in which one can study steering (and non-locality) in quantum theory and beyond. In this framework, we show that kinds of steering, whether quantum or post-quantum, are directly related to particular families of quantum channels that have been previously introduced by Beckman, Gottesman, Nielsen, and Preskill [Phys. Rev. A 64, 052309 (2001)]. Utilising this connection we also (i) demonstrate new analytical examples of post-quantum steering, (ii) give a quantum channel interpretation of almost quantum non-locality and steering, (iii) easily recover and generalise the celebrated Gisin-Hughston-Jozsa-Wootters theorem, and (iv) initiate the study of post-quantum Buscemi non-locality and non-classical teleportation. In this way, we see post-quantum non-locality and steering as just two aspects of a more general phenomenon.

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Quantum mechanics allows for distant systems to be entangled, that is, correlated in a way that admits no equivalent in classical physics. The strongest demonstration of this phenomenon is quantum nonlocality [1, 2]. Performing well-chosen local measurements on separated entangled quantum systems, allows one to observe correlations stronger than in any physical theory satisfying a natural notion of locality, as discovered by Bell. A third form of quantum inseparability is Einstein-Podolsky-Rosen (EPR) steering, which captures the fact that by making a measurement on half of an entangled pair, it is possible to remotely 'steer' the state of the other half. First discussed by Schrödinger [3], this notion was extensively studied in the context of quantum optics [4]. Following a quantum information approach, the concept was put on firm grounds only a few years ago [5], and has attracted growing attention since then. The detection [6, 7] and quantification [8, 9] of steering have been discussed. The concept was also shown to be relevant in quantum information [10, 11], and related to fundamental aspects of quantum theory such as incompatibility of measurements [12, 13]. The phenomena of non-locality and steering are today viewed as fundamental aspects of quantum theory. Hence a deeper understanding of them provides a fresh perspective on the foundations of physics.

Even though these phenomena arise naturally within quantum mechanics, they are not restricted to it. Non-local correlations and steering beyond what quantum theory allows are conceivable while still complying with natural physical assumptions, such as relativistic causality [14, 15]. By "post-quantum"

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we mean non-locality or steering that cannot be realised with local measurements made on an entangled quantum state[1]. Post-quantum non-locality has been widely explored, especially its implications in information-theoretic tasks [16]. Little is known, however, about post-quantum steering, mainly due to the lack of a clear formalism for studying this phenomenon beyond quantum theory.

To rectify the lack of a clear formalism for post-quantum steering, we develop a framework based on the study of quantum channels in which one can study steering (and non- locality) in quantum theory and beyond. The framework is based on the connection between classes of steering and particular families of quantum channels that have been previously introduced by Beckman, Gottesman, Nielsen, and Preskill [17]. Its key feature is the possibility to understand not only quantum but also almost-quantum[2] steering [15] in terms of quantum channels. These analytical constructions of post-quantum steering might be crucial for future explorations on the characterisation of quantum phenomena as well as on the information-theoretical possibilities that post-quantum phenomena may enable. Finally, our formalism also covers the less traditional scenarios of Buscemi-type non-locality [18] as well as non-classical teleportation [19], enabling for the first their generalisation to post-quantum setups. This opens the door to the exploration of where the post-quantum versions of these phenomena could lead us, and further integration of the study of post-quantumness with the study of channels in quantum information theory.

In what follows, we briefly illustrate our methods for the phenomenon of steering.

Steering in quantum and general theories

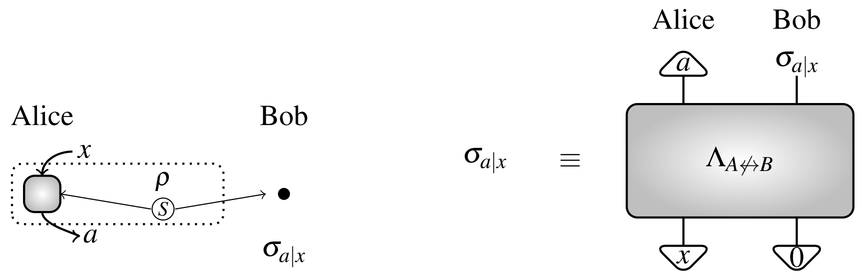
Within quantum theory, a steering scenario is defined as follows (see Fig. 1(a)). For simplicity, in this abstract, we consider the case of a bipartite scenario. Here, two distant parties, Alice and Bob, share a physical system in a quantum state ρ. Moreover, we assume that (i) the actions that the parties make on their share of the system are space-like separated, and (ii) Bob's system and his actions within his lab can be characterised via the quantum formalism. Alice, moreover, by performing a measurement x on her system "steers" that on Bob's lab into the state $\sigma_{a|x}$, where a is the measurement outcome she obtains. If we denote by $\{M_{a|x}\}$ her measurement operators, then $\sigma_{a|x} = \mathrm{tr}_A \left\{M_{a|x} \otimes \mathbb{I}_B \, \rho\right\}$, and the probability with which such a state is prepared is given by $p(a|x) = \mathrm{tr}\{\sigma_{a|x}\}$. The object of study in a steering scenario then is the collection of conditionally prepared states $\{\sigma_{a|x}\}_{a,x}$, a.k.a. assemblage.

To understand steering beyond the formalism of quantum theory one needs to understand the phenomenon in an operational manner, without presuming the validity of quantum theory. In particular, the relaxation that we consider is that where, regardless of the Nature of the underlying physical theory, ultimately (i) Alice's share of the experiment is still characterised by the classical labels of her measurements and outcomes, and (ii) Bob's share of the system is still characterised by a quantum description. Hence, one then takes a step back from the full quantum description of the steering experiment, and merely characterises the scenario by studying assemblages $\{\sigma_{a|x}\}_{a,x}$ which are compatible with the operational constraints of the No-Signalling principle, i.e. the sum $\sum_a \sigma_{a|x}$ is independent of the value of x. Examples of assemblages that are compatible with these operational constraints yet cannot be prepared with said quantum resources have been found in Ref. [15] for tripartite steering scenarios.

The framework we develop in this work then reinterprets the steering scenario mathematically by considering (see Fig. 1(b)) a bipartite quantum channel Λ, with two input ports (one for Alice and one or Bob) and similarly two output ports, such that (i) the state $\sigma_{a|x}$ is seen as the output in Bob's port,

[1]We do not mean post-quantum in the sense of post-quantum cryptography, where one designs cryptographic protocols that cannot be efficiently broken by quantum computers. Post-quantum in our sense could refer to non-locality and steering in generalised probabilistic theories.

[2]Almost quantum steering is seen as a generalisation to steering scenarios of the notion of almost quantum correlations [20].



(a) A quantum steering scenario.

(b) An assemblage $\sigma_{a|x}$ seen as generated by a causal quantum channel $\Lambda_{A\not\leftrightarrow B}$.

Figure 1: **The phenomenon of steering.** (a) Traditional quantum setup. (b) A channel-based reinterpretation for steering within and beyond quantum theory.

whenever (ii) Alice inputs $|x\rangle$ in her input port, and projects the outcome system of her output port in the state $|a\rangle$. Formally, $\sigma_{a|x} = \mathrm{tr}_A \left\{ |a\rangle\langle a| \otimes \mathbb{I}_B \, \Lambda[|x\rangle\langle x| \otimes |0\rangle\langle 0|] \right\}$, where the trace is taken over Alice's input and output Hilbert spaces.

Now, for the assemblage to comply with the operational constraints, the channel Λ should be causal in the sense of [17] between Alice and Bob, and vice versa. Moreover, the assemblage is realisable via the standard quantum steering setup if and only if such channel is a localisable one. Localisable channels define a strict subset of the causal ones, hence by exploring different families of channels in between them we can learn about different flavors of post-quantum steering. In particular, we defined the class of almost-quantum channels, which we prove are linked to the notion of almost-quantum assemblages, a type of post-quantum phenomenon crucial for the problem of characterising quantum behaviors from basic principles [20].

Final remarks

A natural question that may come to mind is why one would be interested in steering in theories beyond quantum theory, since it is a phenomenon that is defined within the quantum formalism. Indeed, if we are testing quantum theory against all possible, sensible classical descriptions of reality, a local hidden variable is the most general starting point. One may however ask in which sensible ways Nature may differ from a world described by quantum theory. Here we argue that it makes sense to consider the picture where locally in our own laboratory everything is described according to quantum theory, however, the global process governing the interactions between laboratories is not, analogous to the study of indefinite causal order in Ref. [21]. The existence of post-quantum steering demonstrates that the global theory can deviate from quantum theory in intriguing ways, even if our own laboratory is restricted to quantum theory.

Our work provides the first playground where post-quantum steering, its properties and information theoretical consequences may be explored. Moreover, not just steering, but also Bell and Buscemi nonlocality, as well as nonclassical teleportation, in an analytical and unified manner. We believe that this approach may shed light on the problem of characterising quantum steering from basic physical principles and of understanding the possibilities and limitations of these nonclassical phenomena in Nature.

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