Efficient Classical Simulation of Some Quantum Algorithms

Niklas Johansson* & Jan-Åke Larsson
Linköping University, Sweden
Links to our contributions can be found in citations [1] and [2]

Abstract

A long-standing aim of quantum information research is to understand what gives quantum computers their advantage. Such an understanding would be of great benefit when attempting to build a quantum computer. Here we present a framework that uses classical resources but still is able to efficiently run, for example Deutsch-Jozsa and Simon's algorithms, and also can run Shor's factoring algorithm with some systematic errors. We also show this explicitly by factoring 15 using classical pass-transistor logic, with smaller systematic errors than any former experimental implementation, and the same amount of resources in time and space as a scalable quantum computer. Our results give further insight into the resources needed for quantum computation.

The idea for studying resources for quantum computation is that if we can learn from what kind of source quantum computers get their advantages, we could focus our effort to get that source to work properly. And the hope is that this would help out in building high-fidelity and scalable quantum computers. There have been many proposals for what these resources could be, some of which are interference [3], entanglement [4], nonlocality [5], contextuality [6, 7, 8], and even coherence [9, 10]. Even tough it seams reasonable that all of these should be present in a universal machine able to simulate the whole of quantum theory, maybe only a few are responsible for the advantage seen from a specific algorithm. If so, that information could be of help in the sens that: if we focus on getting this property to work, the computer will be good at performing this kind of tasks.

From our perspective contextuality was of particular interest, and we decided to search for evidence that this could be the source for the speed-up seen for the Deutsch-Jozsa problem [11]. We were not able to find any such evidence. Instead we ruled it out by simulating the algorithm in an ontological model that can reproduce many quantum phenomena, but cannot produce nonlocal or

*niklas.johansson@liu.se

contextual correlations. More specifically, the model is an extension of Spekkens' model [12] and is described in (Johansson 2017 [1]).

In turn, the model can be efficiently simulated on a classical probabilistic Turing machine, which shows that there is no quantum resources needed to obtain the speed-up.

Similar results (also reported in Johansson 2017 [1]) soon followed for Simon's problem [13]. This raised the question of whether comparing quantum with classical query complexity can give us any conclusive evidence for an advantage at all. In black box query complexity we are given access to an oracle, which given an input returns the output, nothing more nothing less. This is not the case in quantum query complexity. Here we are instead given access to the specific unitary transformation

$$|x\rangle\,|y\rangle \mapsto |x\rangle\,|y \oplus f(x)\rangle\,. \tag{1}$$

From this identity one can be led to believe that what is important is that the unitary adds (mod 2) the function value to the output register. But what is really important is that it preserves the relative phases $c_{x,y}$

$$\sum_{x,y} c_{x,y}\,|x\rangle\,|y\rangle \mapsto \sum_{x,y} c_{x,y}\,|x\rangle\,|y \oplus f(x).\rangle \tag{2}$$

And if we get oracle access to this transformation we can chose to retrieve other information about the function, not only its value for a certain input. But for this information to be available, the oracle needs to obey under an exponential number of constraints; the preservation of an exponential number of amplitudes. Even though it is believed that a fault tolerant construction will be able to handle this, the two oracles are so vastly different that it is reasonable question whether they are comparable at all. And when we compare with a classical framework that aim to simulate this, there is no longer any separation for the Deutsch-Jozsa and Simon's problem.

In the light of this we moved our focus onto Shor's algorithm [14], which does not rely on having access to an oracle. So far we have only manage to approach the problem for small instances, and the statistics returned from our model show that there are deviations from what quantum theory predicts [2]. One can assume that these deviations, or systematic errors, will propagate and amplify as the system grows, however, they are smaller than any current quantum implementations that we know of.

We believe that by understanding these systematic errors we can learn more about the properties that are hard to simulate classically, and thereby learning more about the properties that enables the advantage of quantum computers.

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