Validitiy in Choice Logics

A Game-Theoretic Investigation

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Choice logic = Truth + Preferences

$(a \vee b \vee c) \wedge \neg(a \vee b)$

Choice logic = Classical Logic + Preferences

$F \vec{\times} G$… F or G, but preferably F

$t \wedge (m \vec{\times} a)$

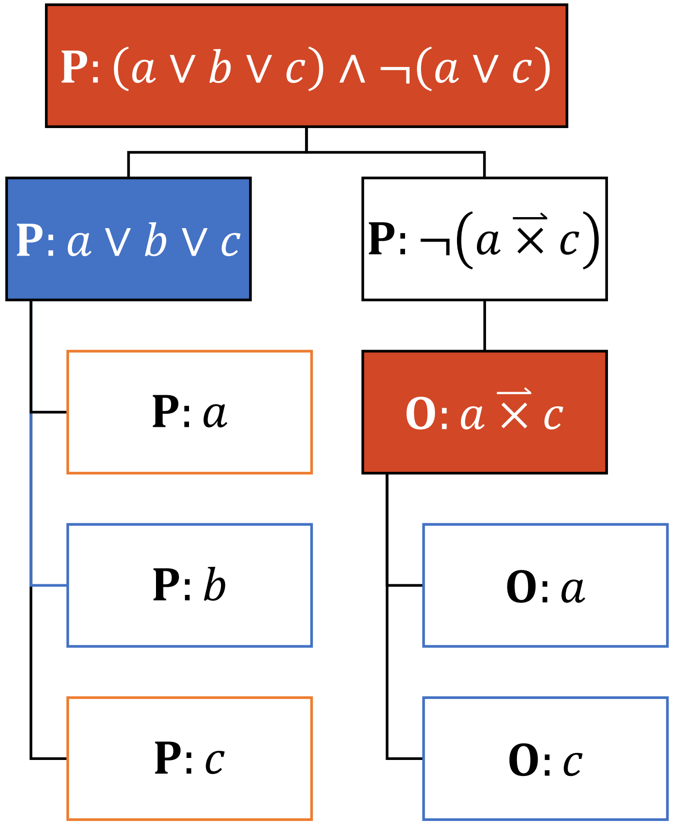
$(a \vec{\times} b \vec{\times} c) \wedge \neg(a \vec{\times} b)$

Hintikka's Semantic Game

- Game between two players, *I* and *You*
- Played over interpretation $\mathcal{I} \subseteq \mathrm{Var}$ and formula F

$F_1 \vee F_2$	Proponent's choice: continue with F_1 or with F_2
$F_1 \wedge F_2$	Opponent's choice: continue with F_1 or with F_2
$\neg G$	game continues with G and a role switch
p	Proponent wins if $p \in \mathcal{I}$ Opponent wins if $p \notin \mathcal{I}$

Example: $\mathcal{I} = \{b\}$



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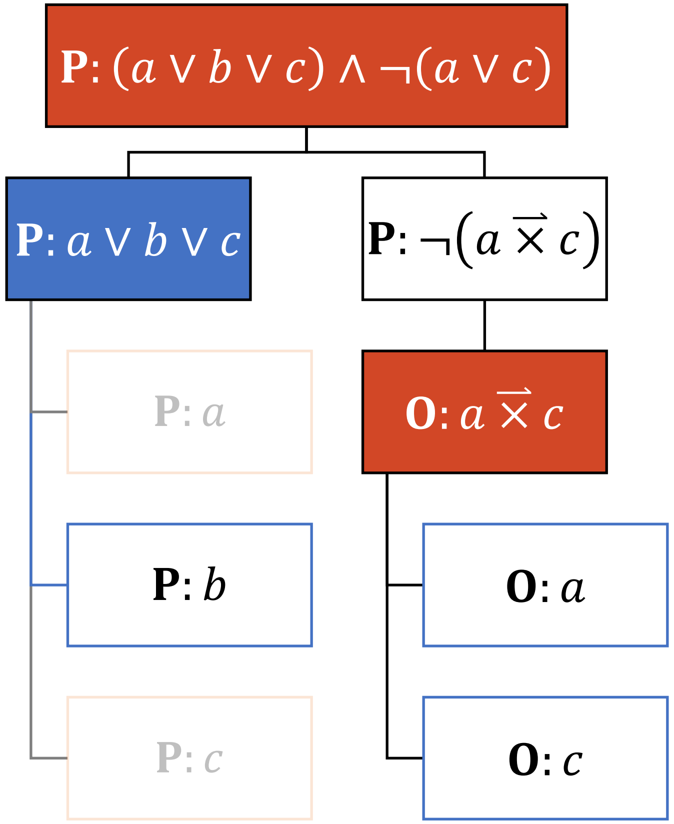
Theorem: *I* have a ws in $\mathrm{G}_{\mathcal{J}}(\mathbf{P}: F)$ iff $\mathcal{J} \vDash F$.

Example: $\mathcal{J} = \{b\}$



Semantic Game for Choice Logics

- Game between two players, *I* and *You*
- Played over interpretation $\mathcal{I} \subseteq \mathrm{Var}$ and formula F

$F_1 \overrightarrow{\times} F_2$ Proponent's choice: continue with F_1 or with F_2
But: this player prefers F_1

→ *Induces preference relation* $\ll$ *on outcomes*

$\delta_{\mathcal{I}}(\mathrm{o})$ P receives payoff $|\pi_{\ll}(o)|$ if they win
P receives payoff $-|\pi_{\ll}(o)|$ if they lose

→ *Payoffs in domain* $Z = \mathbb{Z}^+ \cup \mathbb{Z}^-$

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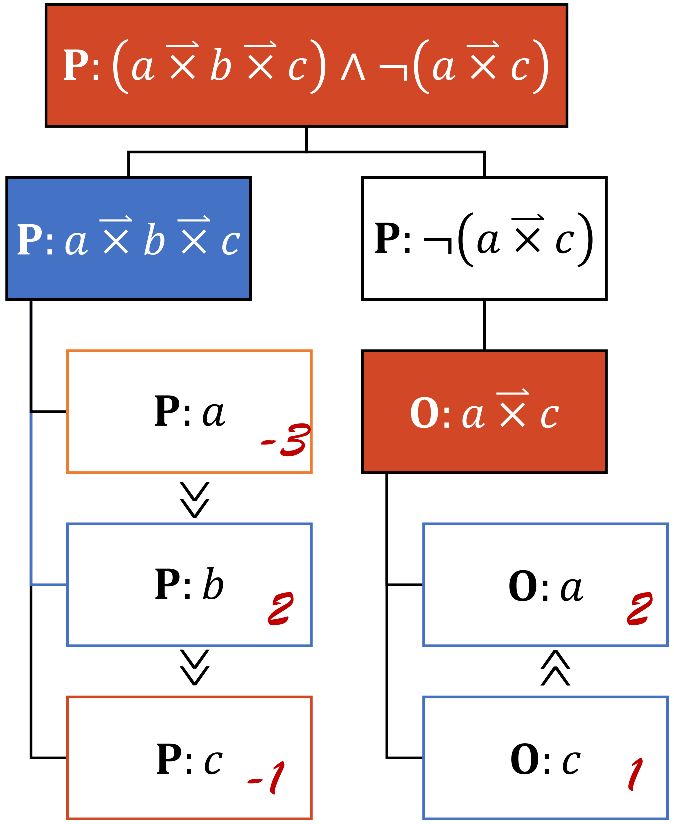
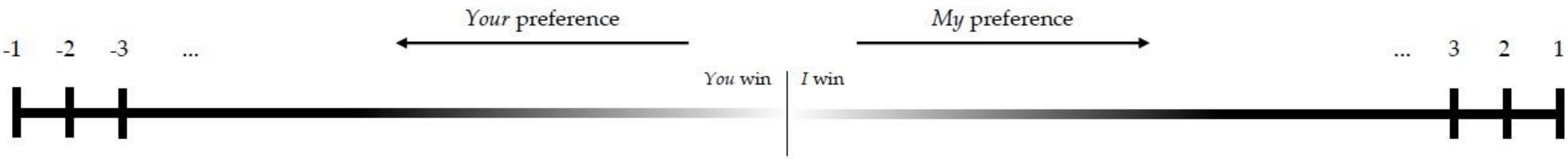


For an outcome o, $|\pi_{\ll}(o)|$ denotes the length of the longest $\ll$-path starting in o



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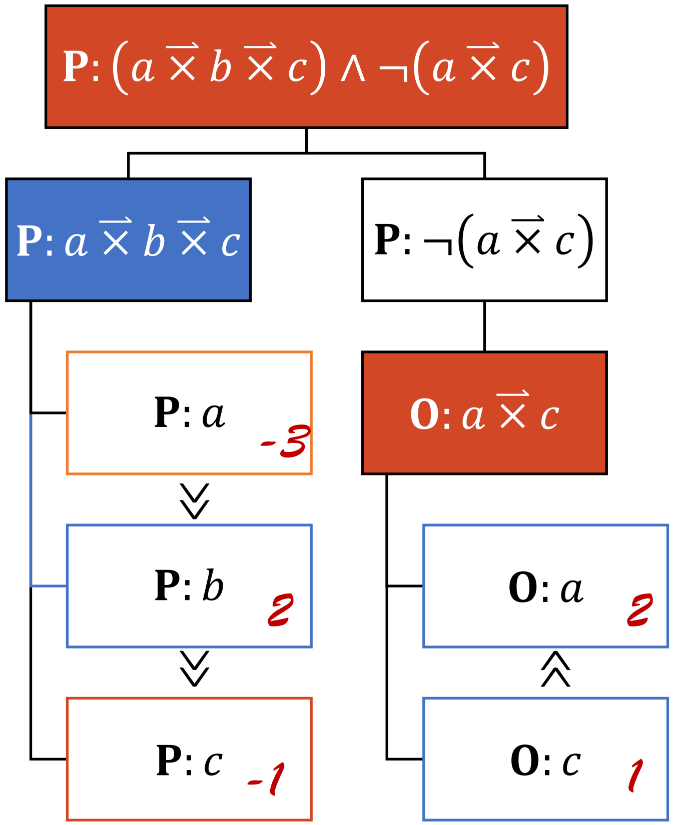
Example: $\mathcal{J} = \{b\}$



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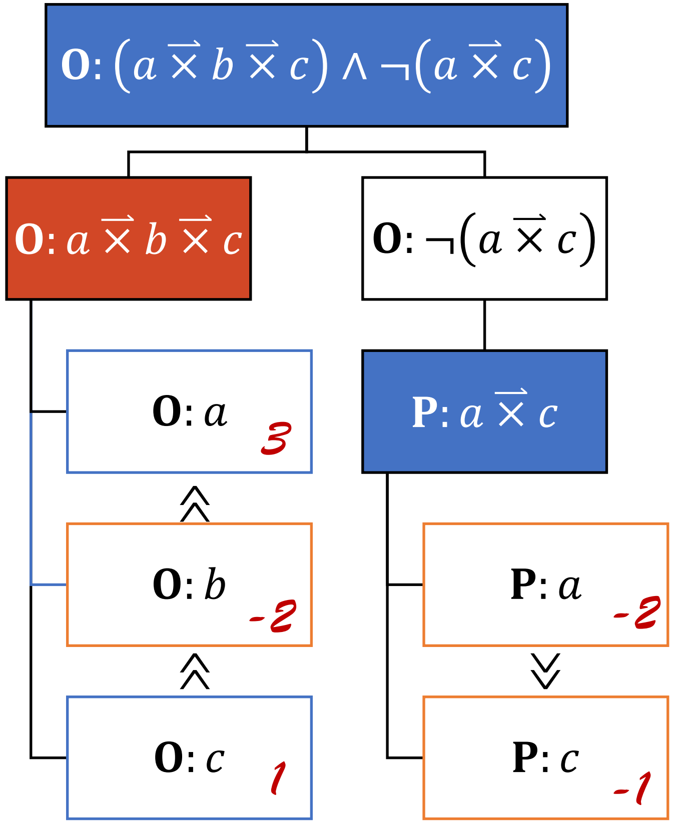
Strategies, Value and Validity

Example: $\mathcal{I} = \{b\}$



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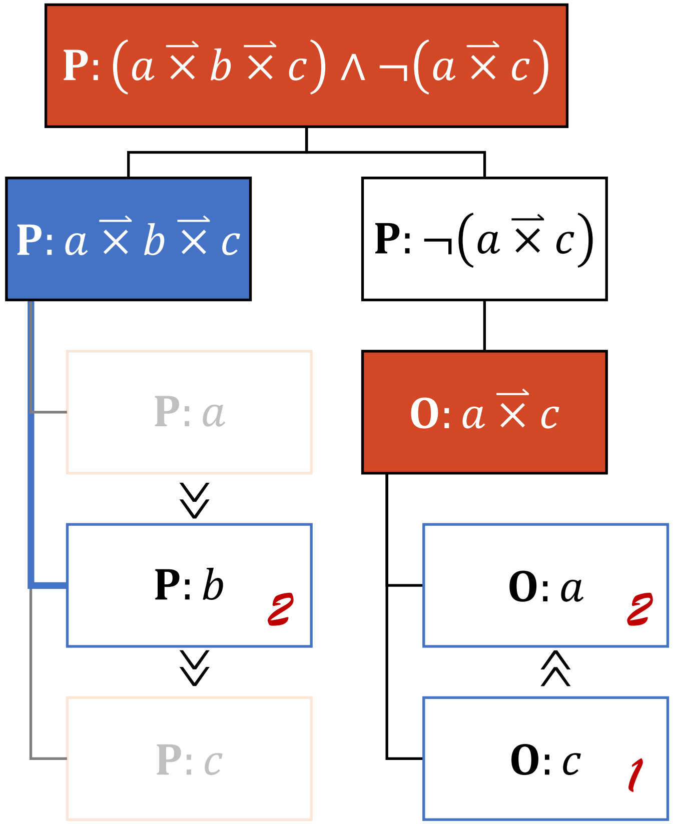

→ 2-strategy!

I/You

Strategies, Value and Validity

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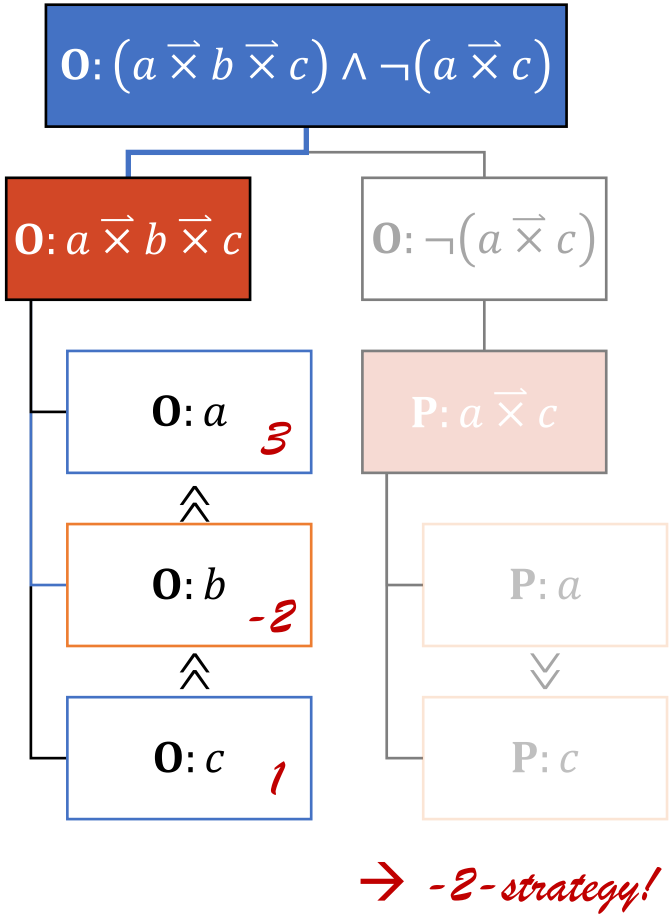

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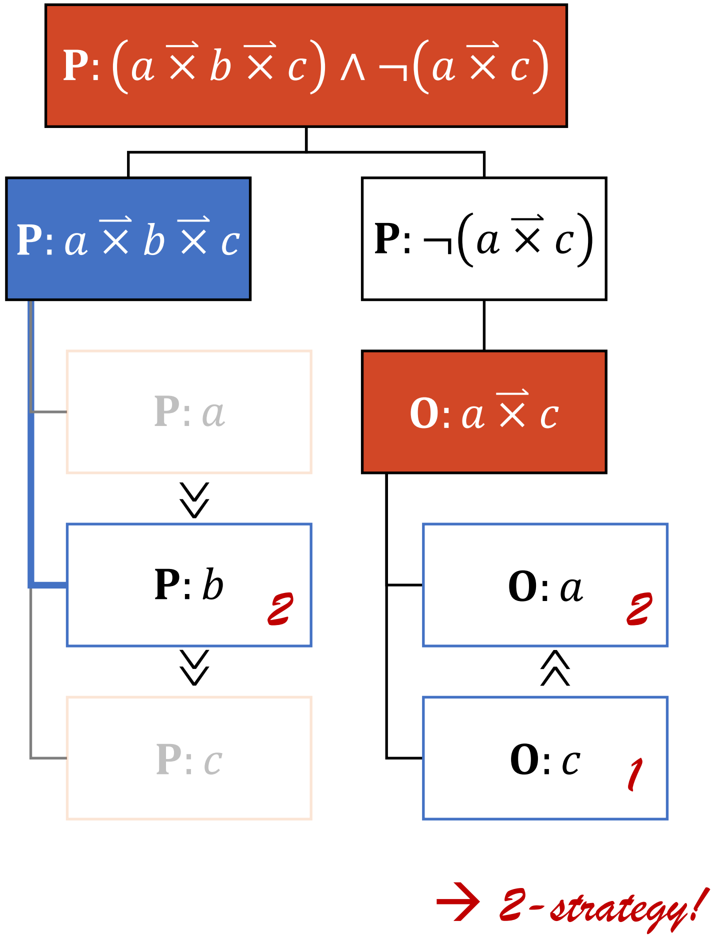

Strategies, Value and Validity

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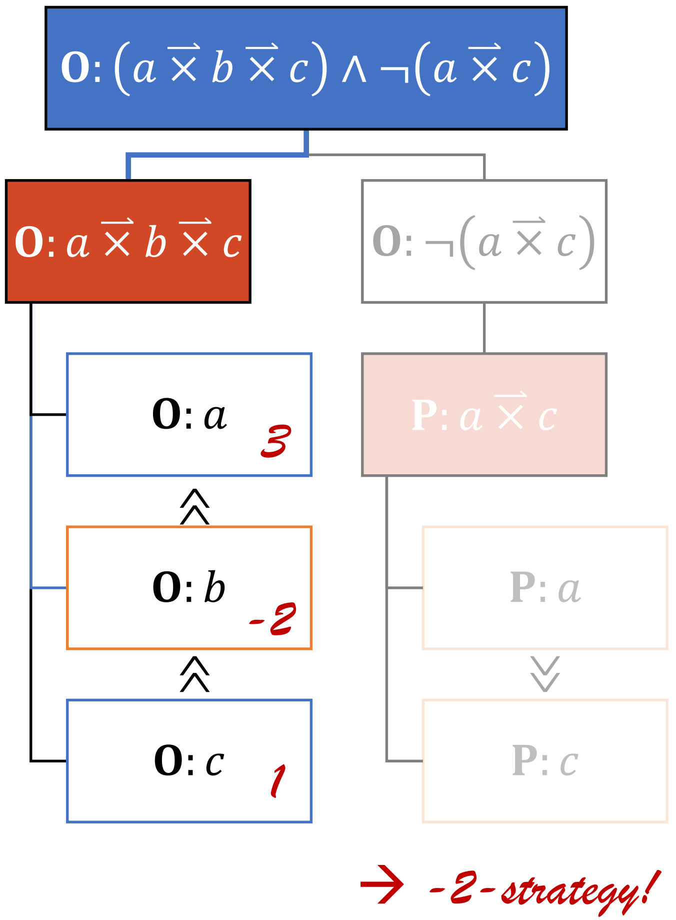


Definition:

$v_{\mathcal{J}}(\mathbf{Q}: F)$ Value/degree of $\mathbf{Q}: F$ = The maximal k such that I have a k-strategy in $\mathbf{G}_{\mathcal{J}}(\mathbf{Q}: F)$

$v(F) = \min_{\mathcal{J}} v_{\mathcal{J}}(\mathbf{P}: F)$ Degree of validity of F

A Game for Validity

Provability Game

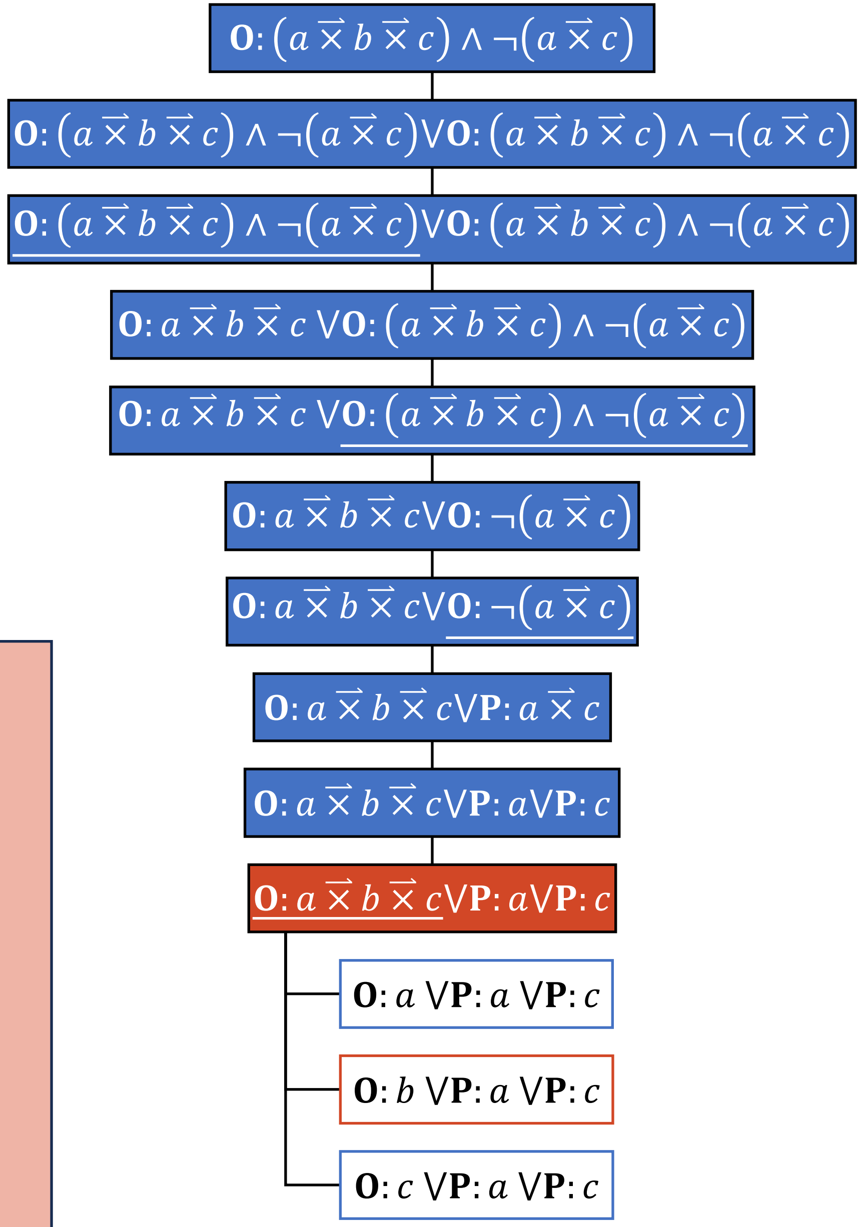
Idea: play over all models simultaneously

But: *I* can create back-up copies

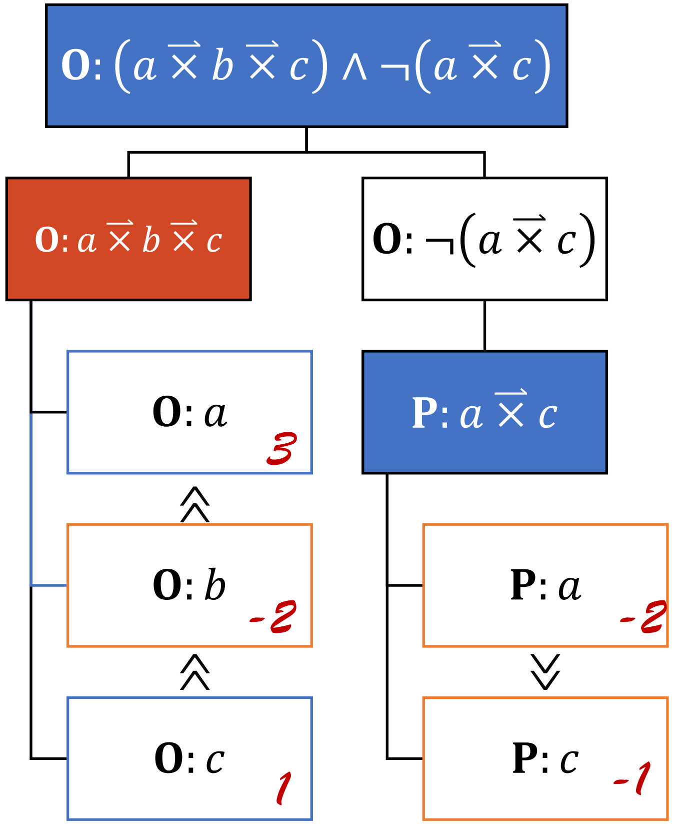
→Game played over disjunctive states $D = \mathbf{Q}_1: F_1 \vee \ldots \vee \mathbf{Q}_n: F_n$

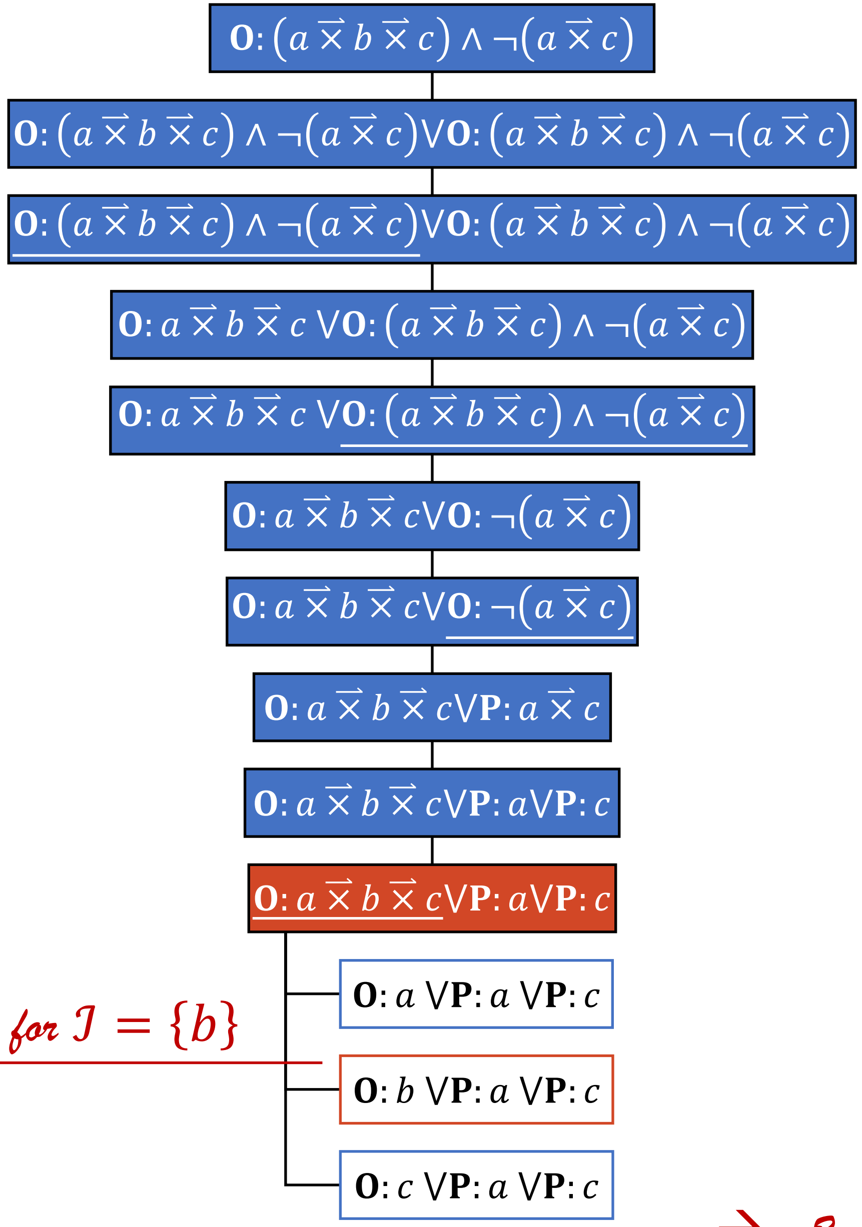
D	*I* choose i, game continues at $\mathbf{Q}_1: F_1 \vee \ldots \vee \underline{\mathbf{Q}_i: F_i} \vee \ldots \vee \mathbf{Q}_n: F_n$ or *I* choose i, game continues at $D \vee \mathbf{Q}_i: F_i$
$D \vee \underline{\mathbf{Q}_i: F_i}$	I/You move to $\mathbf{Q}_i': F_i'$, according to rules of semantic game. New state is $\mathbf{Q}_1: F_1 \vee \ldots \vee \mathbf{Q}_i': F_i' \vee \ldots \vee \mathbf{Q}_n: F_n$

$$\delta(D) = \min_{\mathcal{J}} \max_{o \in D} \delta_{\mathcal{J}}(o)$$



Provability Game





Minimal payoff for $\mathcal{J} = \{b\}$

$$\delta(\mathbf{O}: b \vee \mathbf{P}: a \vee \mathbf{P}: c) = \min_{\mathcal{J}} \max\{\delta_{\mathcal{J}}(\mathbf{O}: b), \delta_{\mathcal{J}}(\mathbf{P}: a), \delta_{\mathcal{J}}(\mathbf{P}: c)\} =$$

$$\max\{\delta_{\{b\}}(\mathbf{O}: b), \delta_{\{b\}}(\mathbf{P}: a), \delta_{\{b\}}(\mathbf{P}: c)\} = \max\{-2, -2, -1\} = -2.$$

→ -2-strategy!

Adequacy

Theorem: The value of the game $\mathbf{DG}(\mathbf{P}: F)$ is $v(F)$, the degree of validity of F.

Proof Theory

My k-strategy = k-proof

$$\mathbf{O}\colon F_1 \vee \ldots \vee \mathbf{O}\colon F_n \vee \mathbf{P}\colon G_1 \vee \ldots \vee \mathbf{P}\colon G_m$$

is written as

$$F_1, \ldots, F_n \Rightarrow G_1, \ldots, G_m$$

Proof system GS for GCL

Initial Sequents for GS

$\Gamma \Rightarrow \Delta$, where Γ and Δ consist of labeled variables

Structural Rules

$$\frac{\Gamma, {}^k_l F, {}^k_l F \Rightarrow \Delta}{\Gamma, {}^k_l F \Rightarrow \Delta} \ (L_c) \qquad \frac{\Gamma \Rightarrow {}^k_l F, {}^k_l F, \Delta}{\Gamma \Rightarrow {}^k_l F, \Delta} \ (R_c)$$

Propositional rules

$$\frac{\Gamma, {}^k_l F \Rightarrow \Delta \quad \Gamma, {}^k_l G \Rightarrow \Delta}{\Gamma, {}^k_l (F \vee G) \Rightarrow \Delta} \ (L_\vee) \qquad \frac{\Gamma \Rightarrow {}^k_l F, \Delta}{\Gamma \Rightarrow {}^k_l (F \vee G), \Delta} \ (R^1_\vee)$$

$$\frac{\Gamma, {}^k_l F \Rightarrow \Delta}{\Gamma, {}^k_l (F \wedge G) \Rightarrow \Delta} \ (L^1_\wedge) \qquad \frac{\Gamma \Rightarrow {}^k_l G, \Delta}{\Gamma \Rightarrow {}^k_l (F \vee G), \Delta} \ (R^2_\vee)$$

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$$\frac{\Gamma \Rightarrow {}^k_l F, \Delta}{\Gamma, {}^k_l \neg F \Rightarrow \Delta} \ (L_\neg) \qquad \frac{\Gamma, {}^k_l F \Rightarrow \Delta}{\Gamma \Rightarrow {}^k_l \neg F, \Delta} \ (R_\neg)$$

Choice rules

$$\frac{\Gamma, {}^{k}_{l+\mathrm{opt}(G)} F \Rightarrow \Delta \quad \Gamma, {}^{k+\mathrm{opt}(F)}_{l} G \Rightarrow \Delta}{\Gamma, {}^k_l (F \vec{\times} G) \Rightarrow \Delta} \ (L_{\vec{\times}}) \qquad \frac{\Gamma \Rightarrow {}^{k+\mathrm{opt}(G)}_{l} F, \Delta}{\Gamma \Rightarrow {}^k_l (F \vec{\times} G), \Delta} \ (R^1_{\vec{\times}})$$

$$\frac{\Gamma \Rightarrow {}^{k}_{l+\mathrm{opt}(G)} G, \Delta}{\Gamma \Rightarrow {}^k_l (F \vec{\times} G), \Delta} \ (R^2_{\vec{\times}})$$

I/You

Example

$$\frac{{}^{1}_{3}a \Rightarrow {}^{2}_{1}a, {}^{1}_{2}c \qquad {}^{2}_{2}b \Rightarrow {}^{2}_{1}a, {}^{1}_{2}c}{{}^{2}_{1}(a \vec{\times} b) \Rightarrow {}^{2}_{1}a, {}^{1}_{2}c} \; (L_{\vec{\times}}) \qquad {}^{3}_{1}c \Rightarrow {}^{2}_{1}a, {}^{1}_{2}c$$

$$(L_{\vec{\times}}) \quad {}^{1}_{1}((a \vec{\times} b) \vec{\times} c) \Rightarrow {}^{2}_{1}a, {}^{1}_{2}c$$

$$(R^{2}_{\vec{\times}}) \quad {}^{1}_{1}((a \vec{\times} b) \vec{\times} c) \Rightarrow {}^{2}_{1}a, {}^{1}_{1}(a \vec{\times} c)$$

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$$(R_C) \quad {}^{1}_{1}((a \vec{\times} b) \vec{\times} c) \Rightarrow {}^{1}_{1}(a \vec{\times} c)$$

$$(L_{\neg}) \quad {}^{1}_{1}((a \vec{\times} b) \vec{\times} c), {}^{1}_{1}(\neg(a \vec{\times} c)) \Rightarrow$$

$$(L_{\wedge}) \quad {}^{1}_{1}((a \vec{\times} b) \vec{\times} c), {}^{1}_{1}(((a \vec{\times} b) \vec{\times} c) \wedge \neg(a \vec{\times} c)) \Rightarrow$$

$$(L_{\wedge}) \quad {}^{1}_{1}(((a \vec{\times} b) \vec{\times} c) \wedge \neg(a \vec{\times} c)), {}^{1}_{1}(((a \vec{\times} b) \vec{\times} c) \wedge \neg(a \vec{\times} c)) \Rightarrow$$

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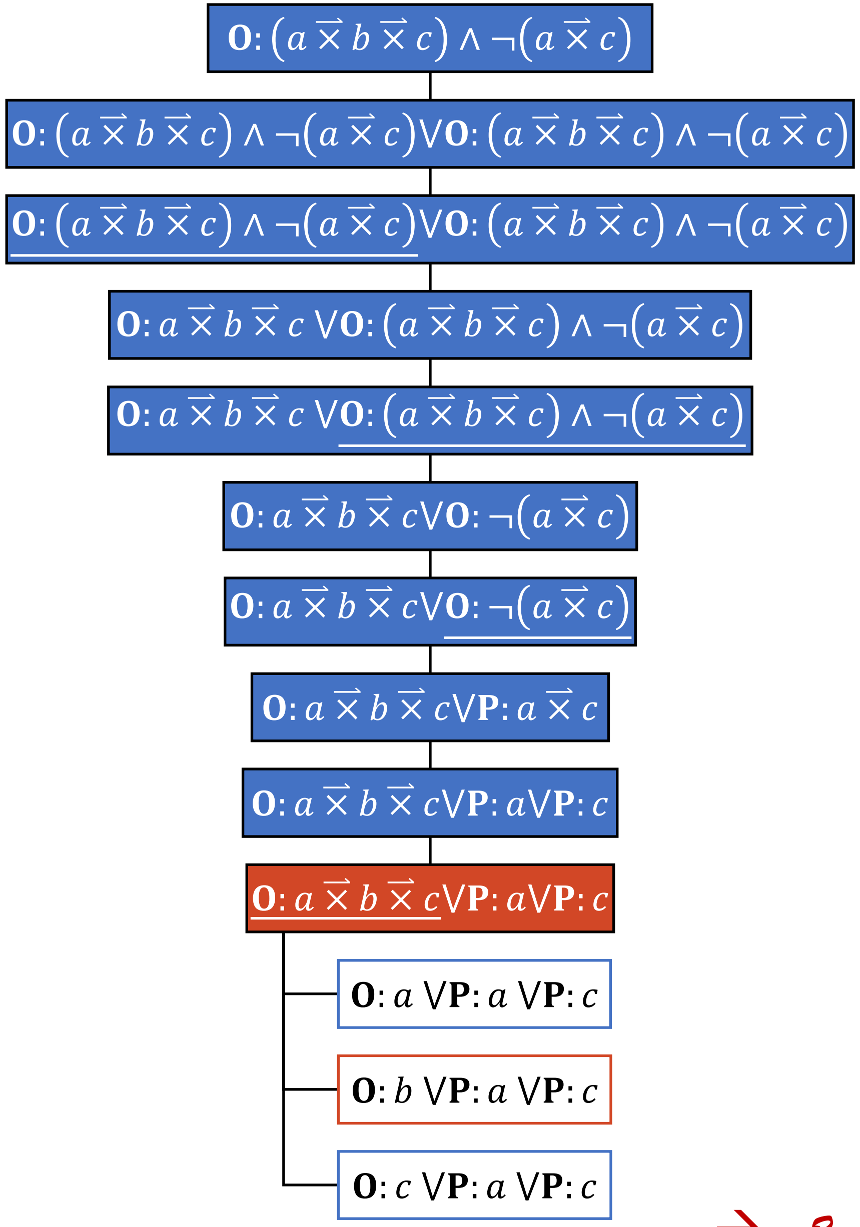
→ -2-proof!



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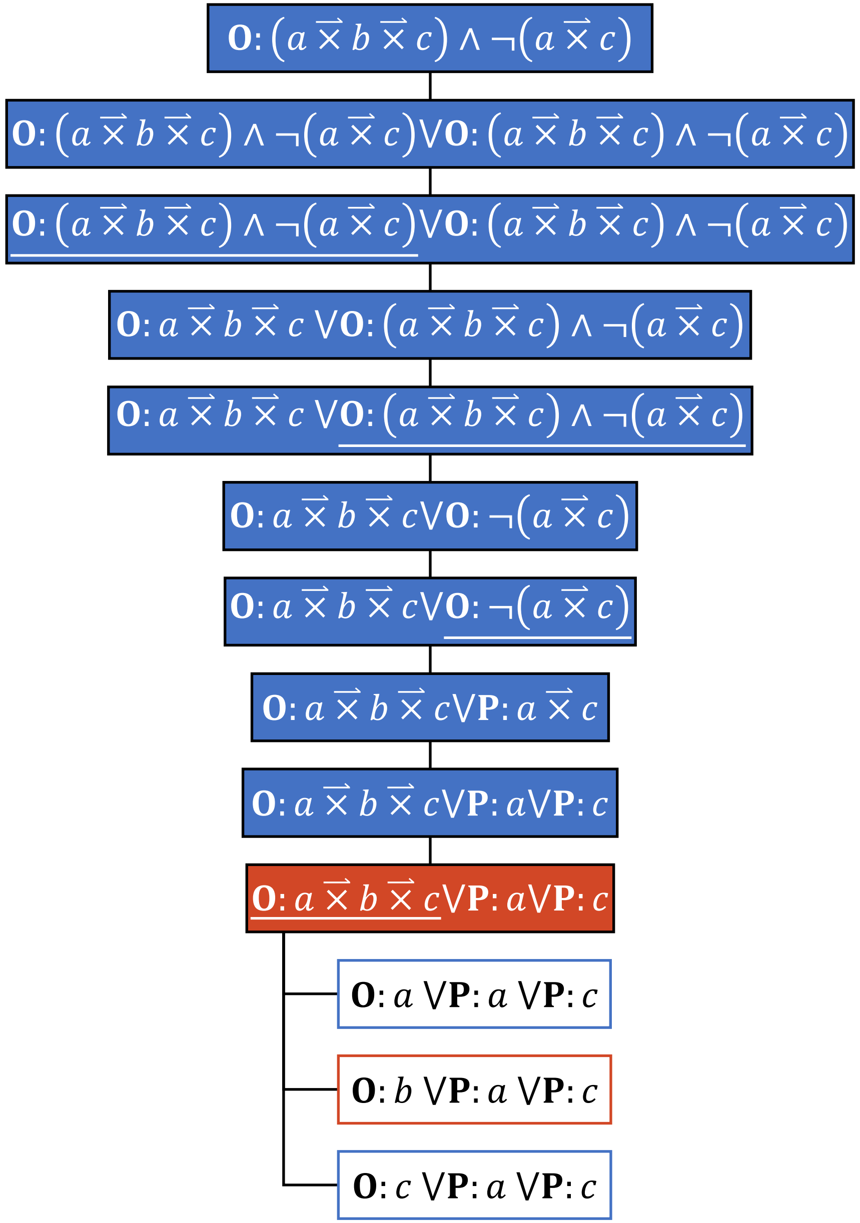
Choice rules

$$\frac{\Gamma, {}^{k}_{l+\mathrm{opt}(G)}F \Rightarrow \Delta \qquad \Gamma, {}^{k+\mathrm{opt}(F)}_{l}G \Rightarrow \Delta}{\Gamma, {}^{k}_{l}(F \vec{\times} G) \Rightarrow \Delta}\ (L_{\vec{\times}})$$

$$\frac{\Gamma \Rightarrow {}^{k+\mathrm{opt}(G)}_{l}F, {}^{k}_{l+\mathrm{opt}(F)}G, \Delta}{\Gamma \Rightarrow {}^{k}_{l}(F \vec{\times} G), \Delta}\ (R_{\vec{\times}})$$

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{{}^1_3a \Rightarrow {}^2_1a, {}^1_2d \qquad {}^2_2b \Rightarrow {}^2_1a, {}^1_2d}{{}^2_1(a\vec{\times}b) \Rightarrow {}^2_1a, {}^1_2d}\,(L_{\vec{\times}}) \qquad {}^3_1c \Rightarrow {}^2_1a, {}^1_2d}{{}^1_1((a\vec{\times}b)\vec{\times}c) \Rightarrow {}^2_1a, {}^1_2d}\,(L_{\vec{\times}})}{{}^1_1((a\vec{\times}b)\vec{\times}c) \Rightarrow {}^1_1(a\vec{\times}d)}\,(R_{\vec{\times}})}{{}^1_1((a\vec{\times}b)\vec{\times}c), {}^1_1\neg(a\vec{\times}d) \Rightarrow}\,(L_{\neg})}{{}^1_1(((a\vec{\times}b)\vec{\times}c) \wedge \neg(a\vec{\times}d)) \Rightarrow}\,(L_{\wedge})}{}$$



Conclusion

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- Game semantics for choice logic
- Negation behaves better
- Same complexity
- Provability game for graded validity
- Sequent-style proof system

Future work

- Preferred model entailment
- Why classical logic?
- Incomparability of outcomes

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Fun Facts

Negation in Choice Logics

	QCL	PQCL	GCL*
degree of $\neg F$ depends only on the degree of F	☑		☑
$F \lor \neg F$ valid	☑	☑	☑
$F \land \neg F$ unsat	☑		☑
F has same degree as $\neg\neg F$		☑	☑
De Morgan's laws	☑	☑	☑
degree semantics	☑	☑	→ SOON

*in GCL, degree=value

Negation in QCL and PQCL

Example: $\neg(a \overrightarrow{\times} b)$ is equivalent to $\neg(a \vee b)$ in QCL and to $\neg a \overrightarrow{\times} \neg b$ in PQCL.

	QCL/PQCL $a \overrightarrow{\times} b$	QCL $\neg(a \overrightarrow{\times} b)$	PQCL $\neg(a \overrightarrow{\times} b)$
$\mathcal{I} = \emptyset$	-1	1	1
$\mathcal{I} = \{a\}$	2	-1	1
$\mathcal{I} = \{b\}$	1	-1	2
$\mathcal{I} = \{a, b\}$	1	-1	-1

Degree semantics QCL

$$
\begin{aligned}
\mathrm{opt}(a) &= 1\\
\mathrm{opt}(\neg F) &= 1\\
\mathrm{opt}(F \wedge G) &= \max\{\mathrm{opt}(F), \mathrm{opt}(G)\}\\
\mathrm{opt}(F \vee G) &= \max\{\mathrm{opt}(F), \mathrm{opt}(G)\}\\
\mathrm{opt}\big(F \overrightarrow{\times} G\big) &= \mathrm{opt}(F) + \mathrm{opt}(G)
\end{aligned}
$$

$$
\begin{aligned}
\deg_{\mathcal{J}}(a) &= \begin{cases} 1 & \text{if } a \in \mathcal{J},\\ -1 & \text{if } a \notin \mathcal{J}. \end{cases}\\
\deg_{\mathcal{J}}(\neg F) &= \begin{cases} 1 & \text{if } \deg_{\mathcal{J}}(\neg F) = -1,\\ -1 & \text{if } \deg_{\mathcal{J}}(\neg F) \in \mathbb{Z}^+. \end{cases}\\
\deg_{\mathcal{J}}(F \wedge G) &= \min_{\trianglelefteq}\{\deg_{\mathcal{J}}(F), \deg_{\mathcal{J}}(G)\}\\
\deg_{\mathcal{J}}(F \vee G) &= \max_{\trianglelefteq}\{\deg_{\mathcal{J}}(F), \deg_{\mathcal{J}}(G)\}\\
\deg_{\mathcal{J}}\big(F \overrightarrow{\times} G\big) &= \begin{cases} \deg_{\mathcal{J}}(F) & \text{if } \deg_{\mathcal{J}}(F) \in \mathbb{Z}^+,\\ \mathrm{opt}(F) + \deg_{\mathcal{J}}(G) & \text{if } \deg_{\mathcal{J}}(F) = -1 \ \deg_{\mathcal{J}}(G) \in \mathbb{Z}^+,\\ -1 & \text{else.} \end{cases}
\end{aligned}
$$

Degree semantics GCL

$$\mathrm{opt}^{\mathcal{G}}(a) = 1$$
$$\mathrm{opt}^{\mathcal{G}}(\neg F) = \mathrm{opt}^{\mathcal{G}}(F)$$
$$\mathrm{opt}^{\mathcal{G}}(F \wedge G) = \max\{\mathrm{opt}^{\mathcal{G}}(F), \mathrm{opt}^{\mathcal{G}}(G)\}$$
$$\mathrm{opt}^{\mathcal{G}}(F \vee G) = \max\{\mathrm{opt}^{\mathcal{G}}(F), \mathrm{opt}^{\mathcal{G}}(G)\}$$
$$\mathrm{opt}^{\mathcal{G}}(F \overrightarrow{\times} G) = \mathrm{opt}^{\mathcal{G}}(F) + \mathrm{opt}^{\mathcal{G}}(G)$$

$$\deg_{\mathcal{J}}^{\mathcal{G}}(a) = \begin{cases} 1 & \text{if } a \in \mathcal{J}, \\ -1 & \text{if } a \notin \mathcal{J}. \end{cases}$$
$$\deg_{\mathcal{J}}^{\mathcal{G}}(\neg F) = -\deg_{\mathcal{J}}^{\mathcal{G}}(F)$$
$$\deg_{\mathcal{J}}^{\mathcal{G}}(F \wedge G) = \min_{\trianglelefteq}\left\{\deg_{\mathcal{J}}^{\mathcal{G}}(F), \deg_{\mathcal{J}}^{\mathcal{G}}(G)\right\}$$
$$\deg_{\mathcal{J}}^{\mathcal{G}}(F \vee G) = \max_{\trianglelefteq}\left\{\deg_{\mathcal{J}}^{\mathcal{G}}(F), \deg_{\mathcal{J}}^{\mathcal{G}}(G)\right\}$$
$$\deg_{\mathcal{J}}^{\mathcal{G}}(F \overrightarrow{\times} G) = \begin{cases} \deg_{\mathcal{J}}^{\mathcal{G}}(F) & \text{if } \deg_{\mathcal{J}}^{\mathcal{G}}(F) \in \mathbb{Z}^+, \\ \mathrm{opt}^{\mathcal{G}}(F) + \deg_{\mathcal{J}}^{\mathcal{G}}(G) & \text{if } \deg_{\mathcal{J}}^{\mathcal{G}}(F) \in \mathbb{Z}^-, \deg_{\mathcal{J}}^{\mathcal{G}}(G) \in \mathbb{Z}^+, \\ \deg_{\mathcal{J}}^{\mathcal{G}}(F) - \mathrm{opt}^{\mathcal{G}}(G) & \text{else.} \end{cases}$$

Theorem: For every F,

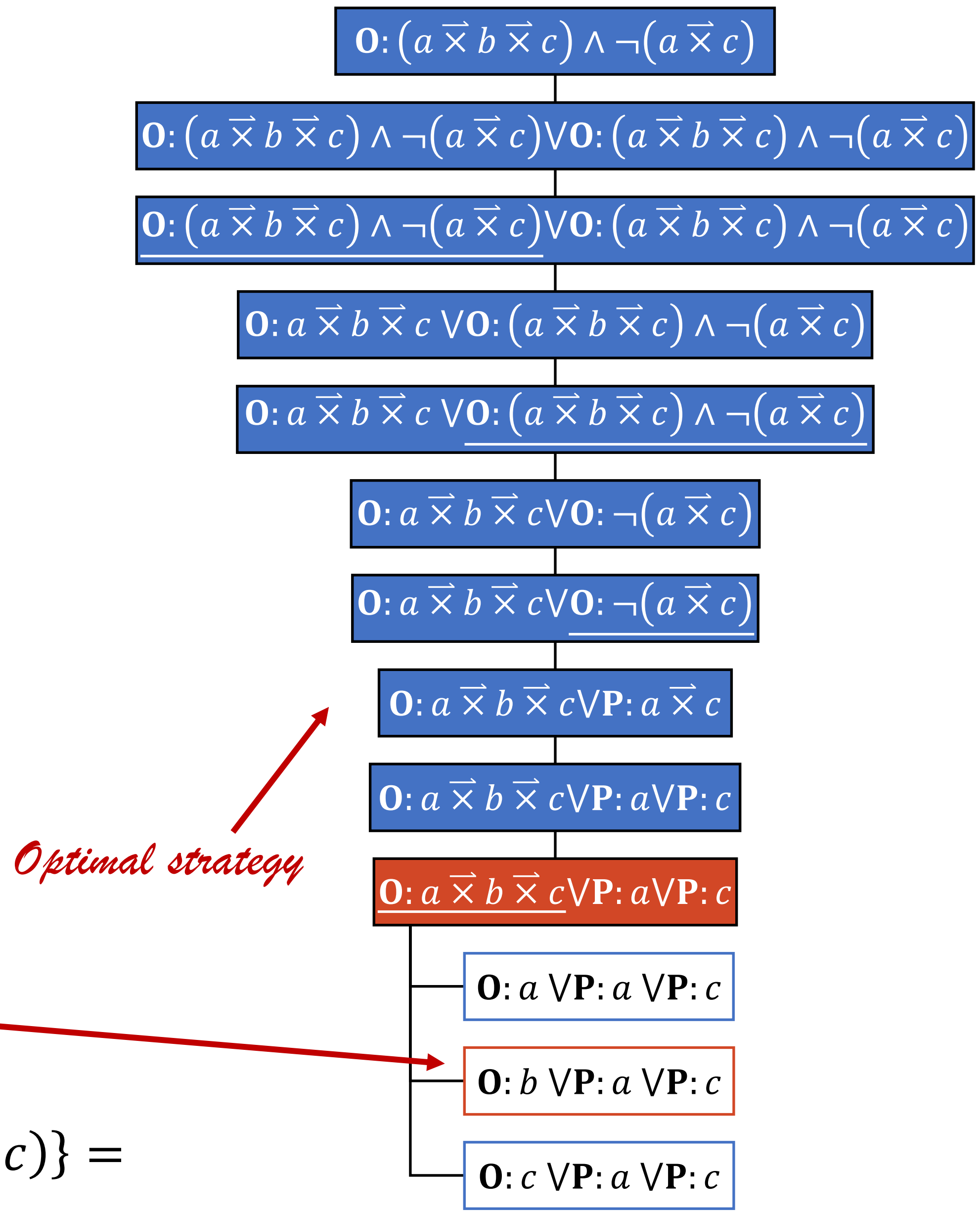
$$v_{\mathcal{J}}(\mathbf{P}: F) = \deg_{\mathcal{J}}^{\mathcal{G}}(F)$$
$$v_{\mathcal{J}}(\mathbf{O}: F) = -\deg_{\mathcal{J}}^{\mathcal{G}}(F)$$

Preferred models

Theorem: Let $\mathcal{I}$ be a preferred model of F and let k be the value of $\mathbf{DG}(\mathbf{O}: F)$. Then $k = -\deg_{\mathcal{I}}^{\mathcal{G}}(F)$ and a preferred model of F can be extracted from *Your* k-strategy or *My* optimal strategy in $\mathbf{DG}(\mathbf{O}: F)$,

Example

$\{b\}$ is a preferred model of $(a \overrightarrow{\times} b \overrightarrow{\times} c) \wedge \neg(a \overrightarrow{\times} c)$ with value 2.



Minimum here

$$\delta(\mathbf{O}: b \vee \mathbf{P}: a \vee \mathbf{P}: c) = \min_{\mathcal{J}} \max\{\delta_{\mathcal{J}}(\mathbf{O}: b), \delta_{\mathcal{J}}(\mathbf{P}: a), \delta_{\mathcal{J}}(\mathbf{P}: c)\} =$$

$$\max\{\delta_{\{b\}}(\mathbf{O}: b), \delta_{\{b\}}(\mathbf{P}: a), \delta_{\{b\}}(\mathbf{P}: c)\} = \max\{-2, -2, -1\} = -2.$$

cut

The following degree-version of cut does not hold:

$$\frac{v(D \vee \mathbf{P}\colon F) \trianglerighteq k \quad v(D \vee \mathbf{O}\colon F) \trianglerighteq k}{v(D) \trianglerighteq k}$$

Example: $v\left(\mathbf{O}\colon \top \vee \mathbf{O}\colon \top \overrightarrow{\times} \bot\right) \trianglerighteq -2$ and $v\left(\mathbf{O}\colon \top \vee \mathbf{P}\colon \top \overrightarrow{\times} \bot\right) \trianglerighteq -2$, but $v(\mathbf{O}\colon \top) = -1$.

cut

There is no function f such that:

$$\frac{v(D \vee \mathbf{P}\colon F) = k \quad v(D \vee \mathbf{O}\colon F) = l}{v(D) = f(k, l)}$$

Example:
$v\left(\mathbf{O}\colon \top \vee \mathbf{O}\colon \top \overrightarrow{\times} \bot\right) = v\left(\vee\, \mathbf{O}\colon \top \overrightarrow{\times} \bot \vee \mathbf{O}\colon \top \overrightarrow{\times} \bot\right) = -2$ and
$v\left(\mathbf{O}\colon \top \vee \mathbf{P}\colon \top \overrightarrow{\times} \bot\right) = v\left(\vee\, \mathbf{O}\colon \top \overrightarrow{\times} \bot \vee \mathbf{P}\colon \top \overrightarrow{\times} \bot\right) = 2$, but

$$v(\mathbf{O}\colon \top) = -1 \neq -2 = v\left(\mathbf{O}\colon \top \overrightarrow{\times} \bot\right)$$

Complexity results

- DegCheckng: given a disjunctive state D and $k \in Z$, is $v(D) \trianglerighteq k$? - coNP-complete
- DegCheckngInit: given an elementary disjunctive state D and $k \in Z$, is $v(D) \trianglerighteq k$? - in P
- PMChecking: given a formula F and an interpretation $\mathcal{I}$, is $\mathcal{I}$ a preferred model of F? - coNP-complete for QCL and GCL
- PMContainment: given a GCL-formula F and a variable a, is there a preferred model $\mathcal{I}$ of F such that $a \in \mathcal{I}$? - Θ_2^{P}-complete for QCL and GCL