From incompleteness of static theories to completeness of dynamic beliefs in people and in bots

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Outline

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Static incompleteness and dynamic completeness

Universality

Static incompleteness

Dynamic completeness

World of states

String diagrams

Universal language

Self-confirming beliefs

Self-updating explanations

Frege, . . . , Tarski: soundness, completeness

$$\begin{tikzcd} \Theta \vdash \alpha \arrow[d, Rightarrow, bend right, "\text{sound}"'] \\ \mathcal{M} \models \alpha \arrow[u, Rightarrow, bend right, "\text{complete}"'] \end{tikzcd}$$

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Gödel: Peano arithmetic is incomplete for $\gamma \vee \neg\gamma$

Gödel's incompleteness claim

$$\text{universal language } \mathbb{L} \quad \Longrightarrow \quad \exists\varphi.\ \mathcal{M} \models \varphi \ \wedge\ \Theta \not\vdash \varphi$$

Belief completeness claim

$$\left.\begin{array}{c}\text{universal language } \mathbb{L} \\ + \\ \text{belief updating}\end{array}\right\} \Longrightarrow \begin{cases} \forall\ X\text{-process } \mathcal{M} \\ \exists\ X\text{-beliefs } \Theta \text{ in } \mathbb{L} \\ \forall \text{ state } x \text{ in } X. \\ \mathcal{M}_x \models \alpha \iff \Theta_{\mathcal{M}_x} \vdash \alpha \end{cases}$$

Belief completeness claim

$$\left.\begin{array}{c}\text{universal language } \mathbb{L} \\ + \\ \text{belief updating}\end{array}\right\} \implies \left\{\begin{array}{c}\forall\, x : X \\ \forall X \xleftarrow{\mathcal{M}_x} A \xrightarrow{[x]-} B \\ \exists\, \Theta_x : \mathbb{L}. \\ \left[\mathcal{M}_x\right]\alpha \models \beta \\ \Updownarrow \\ \left\{\Theta_x\right\}''\alpha \vdash \beta \\ \wedge \\ \left\{\Theta_x\right\}'\alpha = \Theta_{\mathcal{M}_x\alpha}\end{array}\right.$$

Belief completeness claim

$$\left.\begin{array}{c}\text{universal language } \mathbb{L}\\ +\\ \text{belief updating}\end{array}\right\} \Longrightarrow \left\{\begin{tikzcd} X \arrow[d, dashed, "\Theta"'] & X \times A \arrow[l, "\mathcal{M}"'] \arrow[d, dashed, "\Theta \times id"] \arrow[dr, "{[-]-}"] & \\ \mathbb{L} & \mathbb{L} \times A \arrow[l, "\{\}'"] \arrow[r, "\{\}''"'] & B \end{tikzcd}\right.$$

Universal language

is a type $\mathbb{L}$ equipped with for each pair A, B an *interpreter*

$$\big\{\big\} : \mathbb{L} \times A \to B$$

such that for every $f : X \times A \to B$ there is $\ulcorner f \urcorner : X \to \mathbb{L}$ with

$$f_x(a) \quad = \quad \big\{\ulcorner f \urcorner(x)\big\}(a)$$

Intuition

Think of

- the elements of $\mathbb{L}$ as *belief states*[1]
- the belief states $\ulcorner f \urcorner(x) : \mathbb{L}$ as *explanations* of f

[1] e.g., evolving scientific theories

Self-confirming beliefs

Every believer $g \colon \mathbb{L} \times A \to B$ has a self-confirming belief $\Gamma \colon \mathbb{L}$ with

$$g(\Gamma, a) \quad = \quad \{\Gamma\}\,(a)$$

Static incompleteness proof

- set $\mathbb{L} = \mathbb{N}$ with arithmetic.

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- soundness implies
$$\P(\ulcorner p \urcorner, a) = \{\ulcorner p \urcorner\}(a) = p(a) \qquad (\ddagger)$$

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- soundness implies
$$\P(\ulcorner p \urcorner, a) = \{\ulcorner p \urcorner\}(a) = p(a) \tag{$\ddagger$}$$
- instantiate $g(p, a) = \P(\ulcorner \neg p \urcorner, a)$ to get $\gamma \colon A \to \mathbb{B}$ with
$$\P(\ulcorner \neg\gamma \urcorner, a) \stackrel{(*)}{=} \gamma(a) \stackrel{(\ddagger)}{=} \P(\ulcorner \gamma \urcorner, a)$$

Static incompleteness proof

Since $\P(\ulcorner p \urcorner, a) \iff \vdash p(a)$, this means

$$\vdash \neg\gamma \iff \vdash \neg\gamma$$

and hence

$$\nvdash \neg\gamma \vee \gamma$$

Static incompleteness proof

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$$\vdash \neg\gamma \quad \iff \quad \vdash \neg\gamma$$

and hence

$$\nvdash \neg\gamma \vee \gamma$$

But in the standard model of arithmetic, all γ satisfy

$$\models \neg\gamma \vee \gamma$$

Dynamic completeness setting

Think of functions $f = \langle f', f'' \rangle : X \times A \to X \times B$ as *processes* over the state space X, with

- $f' : X \times A \to X$ is an X-*state update after* A
- $f'' : X \times A \to B$ is an X-*family of* B-*effects* of A

Self-confirming beliefs of families

Every X-family of believers $t\colon X \times \mathbb{L} \times A \to B$ has an X-family of self-confirming beliefs $\ulcorner t \urcorner : X \to \mathbb{L}$, i.e.

$$\left\{\ulcorner t \urcorner(x)\right\}(a) \quad = \quad t_x\left(\ulcorner t \urcorner(x), a\right) \qquad \boxed{*}$$

Self-updating explanations

Every process $f\colon X \times A \to X \times B$ has
a self-updating explanation $[\![f]\!]\colon X \to \mathbb{L}$, i.e.

$$\underbrace{\left\{[\![f]\!]_x\right\}'(a) = [\![f]\!]\left(f'_x(a)\right)}_{\boxed{\dfrac{f' : x \longmapsto f'_x}{\{\}' : [\![f]\!](x) \longmapsto [\![f]\!]\left(f'_x\right)}}} \qquad \underbrace{\left\{[\![f]\!](x)\right\}''(a) = f''_x(a)}_{\boxed{*}}$$

Outline

Wittgenstein,. . . , Kripke: states

(Sachverhalte, . . . , possible worlds)

A *state* is a model of a theory.

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A model of a theory assigns meanings to its expressions, making its theorems true.

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(Sachverhalte, ..., possible worlds)

A *state* is a model of a theory.

A model of a theory assigns meanings to its expressions, making its theorems true.

If a theory is expressed as a sketch $\Theta = \langle \Sigma, \Gamma \rangle$, then a state is a Γ-preserving functor $\mathcal{M}\colon \Sigma \to \mathrm{Set}$.

States and spaces

Definition

A *state space* is a pair $A = \langle \Theta_A, \mathcal{M}_A \rangle$ where

- $\Theta_A = \langle \Sigma_A, \Gamma_A \rangle$ is a theory, where
  - Σ_A is the signature and
  - Γ_A are the axioms
- $\mathcal{M}_A$ is a reference model of Θ_A, i.e. a Γ_A-preserving functor $\mathcal{M}_A \colon \Sigma_A \to \mathsf{Set}$.

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Remark

The states from a space A are the refinements of $\mathcal{M}_A$. They form an accessible category.

States and spaces

Examples

- Θ = a scientific theory
  $\mathcal{M}$ = settled measurements

States and spaces

Examples

- Θ = a scientific theory
  $\mathcal{M}$ = settled measurements
- Θ = a software specification
  $\mathcal{M}$ = reference implementation

States and spaces

Examples

- Θ = a scientific theory
  $\mathcal{M}$ = settled measurements
- Θ = a software specification
  $\mathcal{M}$ = reference implementation
- Θ = situation (Popper, Barwise)
  $\mathcal{M}$ = observations

States and spaces

Examples continued

- Θ = actor network (Latour et al.)
  $\mathcal{M}$ = protocol

States and spaces

Examples continued

- Θ = actor network (Latour et al.)
  $\mathcal{M}$ = protocol
- Θ = protocol
  $\mathcal{M}$ = session

States and spaces

Examples continued

- Θ = actor network (Latour et al.)
  $\mathcal{M}$ = protocol
- Θ = protocol
  $\mathcal{M}$ = session
- Θ = network layer
  $\mathcal{M}$ = stack interfaces

State transitions

A *state transition* $f: A \to B$ is a pair $f = \langle f_\Theta, f_\Gamma \rangle$ where

- $f_\Gamma : \Gamma_A \leftarrow \Gamma_B$ is a function, whereas
- $f_\Theta : \Theta_A \leftarrow \Theta_B$ is a functor such that

$$\begin{tikzcd}
\Theta_A \arrow[dr, dashed, "\overline{\mathcal{M}}_A"'] & & \Theta_B \arrow[ll, bend right, "f"'] \arrow[dl, dashed, "\overline{\mathcal{M}}_B"] \\
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where

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- $\overline{\mathcal{M}}_A$ is the Γ_A-preserving extension of $\mathcal{M}_A$; ditto $\overline{\mathcal{M}}_B$
- f_Θ maps Γ_B-(co)limits into Γ_A-(co)limits according to f_Γ.

State transitions

Examples

- scientific theory updates
- software specification refinements
- events
- protocol options, suites, stacks, attacks...
- network integration...

Monoidal universe

Let $\mathcal{U}$ be the category of

- state spaces as objects and
- state transitions as morphisms.

with the monoidal product

$$A \otimes B \;=\; \left\langle \Sigma_A + \Sigma_B \,,\, \Gamma_A + \Gamma_B \,,\, [\mathcal{M}_A + \mathcal{M}_B] \right\rangle$$

Monoidal universe with comonoids

Every state A comes with a comonoid

$$A \otimes A \xleftarrow{\;\Delta\;} A \xrightarrow{\qquad} I$$

$$\Sigma_A + \Sigma_A \xrightarrow{[id,id]} \Sigma_A \xleftarrow{\;\perp\;} \perp$$

Cartesian transitions

Let $\mathcal{U}^\bullet$ be the category of

- state spaces and
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Cartesian transitions

Let $\mathcal{U}^{\bullet}$ be the category of

- state spaces and
- comonoid homomorphisms.

Remarks

- $\mathcal{U}^{\bullet}$ is the largest cartesian subcategory of $\mathcal{U}$.
- A transition $f = \langle f_{\Theta}, f_{\Gamma} \rangle : A \to B$ is in $\mathcal{U}^{\bullet}$ if and only if $f_{\Theta} : \Theta_A \leftarrow \Theta_B$ restricts to $f_{\Sigma} : \Sigma_A \leftarrow \Sigma_B$.

Outline

Logical inference

static: Hoare:

$A \vdash B$ $A \left[p \right] B$

Logical inference

static: dynamic:

$A \vdash B$ $\quad\quad$ $\left[p\right] A \models B$

Logical inference

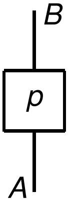
static:	dynamic:	evolving:
$A \vdash B$	$\left[p\right] A \models B$	$q\colon X \times A \to X \times B$

Logical inference in string diagrams

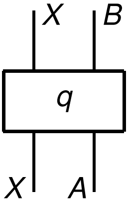
static:

$$\frac{A}{B}$$

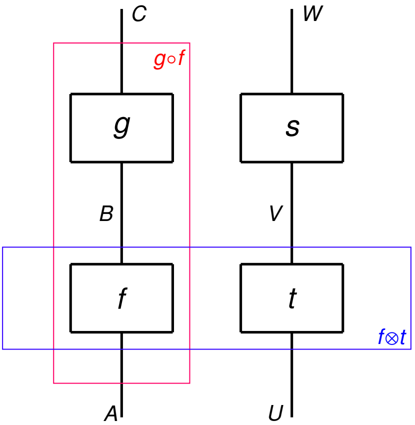
dynamic:



evolving:



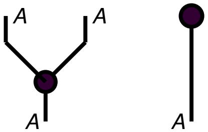
String diagrams are 2-dimensional text



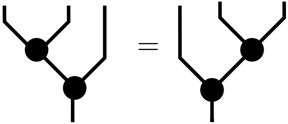
$$(g \circ f) \otimes (s \circ t) = (g \otimes s) \circ (f \otimes t)$$

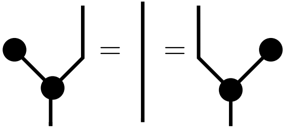
Comonoids are data services

$$A \otimes A \xleftarrow{\;\Delta\;} A \xrightarrow{\quad} I$$



Comonoid equations





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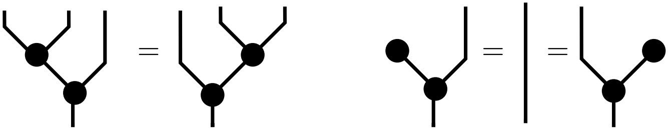
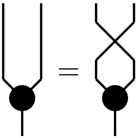
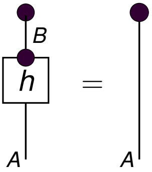
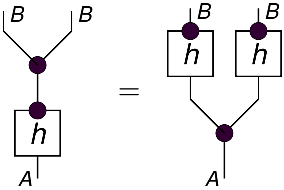


. . . also commutative:



Comonoid homomorphism





Outline

Universal language

is a type $\mathbb{L}$ equipped with for each pair A, B an *interpreter*

$$\{\}: \mathbb{L} \times A \to B$$

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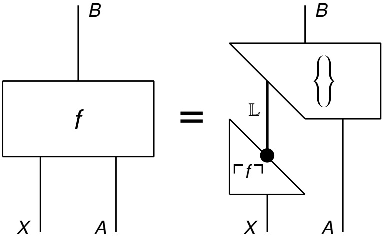


Universal language

Example 1

If the states are $A = \langle \Theta_A, \mathcal{M}_A \rangle$ where

- $\Theta_A = \langle \Sigma_A, \Gamma_A \rangle$ is a sketch of a first-order theory
- $\mathcal{M}_A \colon \Sigma_A \to \mathsf{Set}$ is a Γ_A-preserving functor

then $\mathbb{L} = \langle \Theta_\mathbb{L}, \mathcal{M}_\mathbb{L} \rangle$ is

- $\Theta_\mathbb{L}$ = sketch of state transitions
- $\mathcal{M}_\mathbb{L} = 1$

Universal language

Example 2

If the states *A* are computably enumerable types

- (enumerations = computable projections)

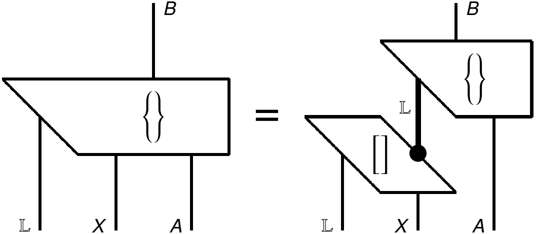
then $\mathbb{L}$ is a universal programming language

- (interpreters = interpreters)

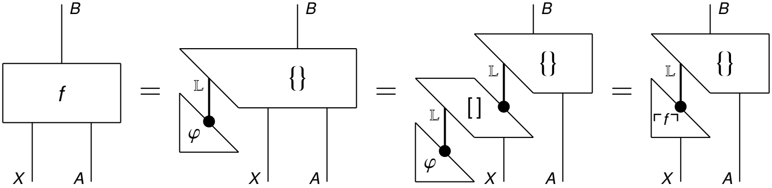
Specializer

Lemma

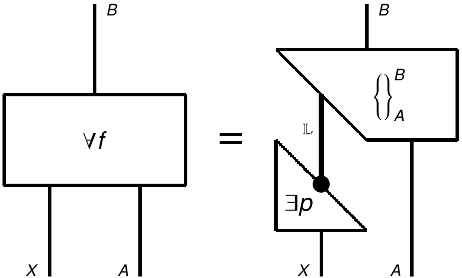
For all $X, A, B \in \mathcal{U}$ there is a strict transition $[]_X^{AB} \in \mathcal{U}^\bullet(\mathbb{L} \times X, \mathbb{L})$ such that



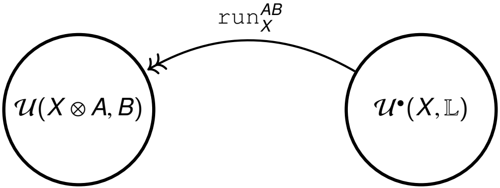
Slicing state changes



Universality = polymorphism



induces program-closure $\mathtt{run}_{\mathbb{P}}^{AB}(p) = \{p\}_A^B$



Single-instruction programming language

Programs as Diagrams

From Categorical Computability to Computable Categories

Dusko Pavlovic

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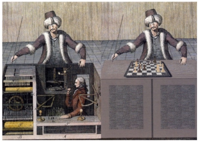

4 What can be computed

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(or dusko.org)

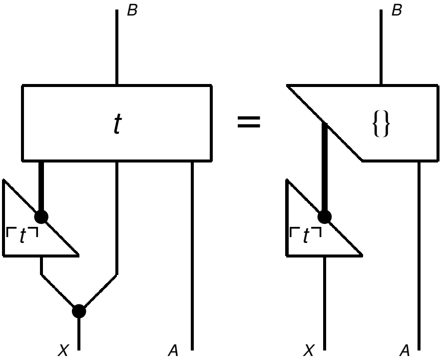
Logic as an evolutionary process

$$\frac{\text{program construction}}{\text{program evaluation}} = \frac{\text{induction}}{\text{deduction}} = \frac{\text{learning}}{\text{prediction}}$$

Outline

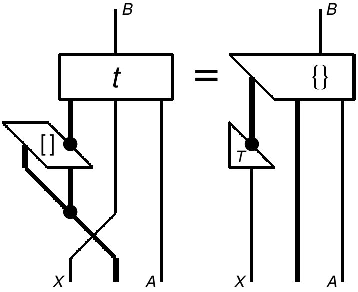
Self-confirming beliefs theorem

Every family of believers $t \in \mathcal{U}(\mathbb{P} \otimes X \otimes A, B)$ has a family of self-confirming beliefs $\ulcorner t \urcorner \in \mathcal{U}^\bullet(X, \mathbb{P})$

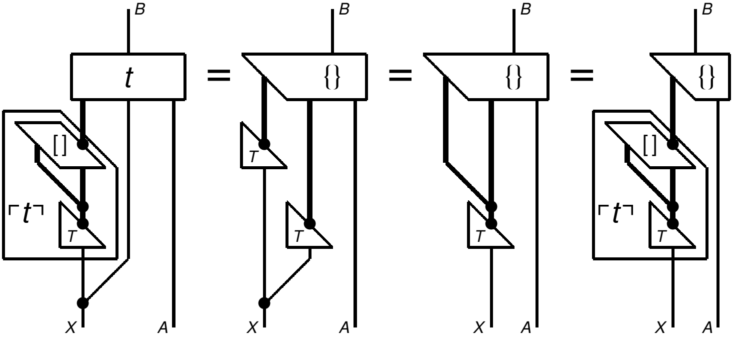


$\boxed{*}$

Self-confirming beliefs construction



Self-confirming beliefs construction



Self-confirming beliefs examples

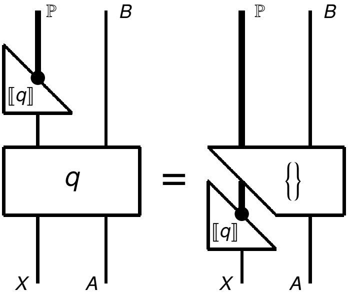
- observer's bias in scientific theories
  - sampling bias (R. Fisher: controls)
  - design bias (J. Neyman: causal inference, allocation)
  - anthropic assumptions (F. Hoyle!)
  - cultural relativism (M. Mead, Levi-Strauss, . . .)
- self-validating software (Knuth, Blum, Rubinsfeld. . .)
- "Credo quia absurdum"
- . . .

Outline

Self-updating explanation theorem

Every process $q\colon X \times A \to X \times B$ has a self-updating explanation $[\![q]\!]\colon X \to \mathbb{L}$, consistent with state changes:

$$[\![q]\!]\,(q'_x(a)) = \left\{[\![q]\!]_x)\right\}'(a) \qquad\qquad q''_x(a) = \left\{[\![q]\!](x)\right\}''(a)$$



Complete beliefs

D. & T. Pavlovic

(In)completeness
World of states
String diagrams
Universal
Self-confirming
Self-updating
Bot beliefs

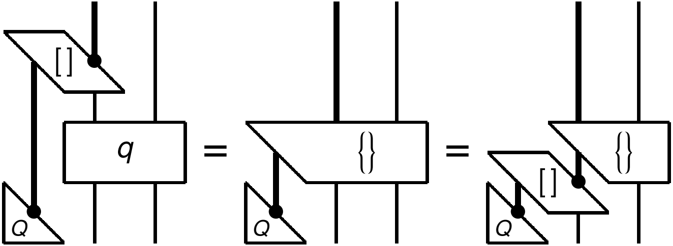
Self-updating explanation **explanation**

Consistency with the state changes means that $[\![q]\!]$ explains q

- not only at the state x by $[\![q]\!]_x$
- but also at the state $q'_x(a)$ by $[\![q]\!]_{q'_x(a)} = \left\{[\![q]\!]_x\right\}'(a)$

for all a in A and x in X.

Self-updating explanation construction



Self-updating explanation example

- X = uses of a social network
  - $u(x) = x$ $\leftsquigarrow$ utility
  - $v(x) = 1 - x$ $\leftsquigarrow$ demand
  - $w(x) = u(x) \cdot v(x) = x - x^2$ $\leftsquigarrow$ valuation
- A = costs of a social network
  - $p =$ price as a supply-demand equilibrium of providers' *influencing* investments and users' *influnced* payments
- B = price update
  - $d =$ price difference

Self-updating explanation example

- adoption process (Brian Arthur)

$$\begin{aligned} q: X \times A &\to X \times B \\ \langle x, p \rangle &\longmapsto \langle x', p' \rangle \\ & \quad x' = x + \delta(p, \tau) \\ & \quad p' = p - d(p, \tau) \end{aligned}$$

where

  - $\tau =$ investment distribution
  - $\delta(p, \tau) =$ part of x believing that $p < x - x^2$
  - $d(p, \tau) =$ adoption discount

Self-updating explanation example

- deployment strategy

$$
\begin{aligned}
[\![q]\!] : X &\to \mathbb{L} \\
x &\longmapsto \begin{cases} \text{advertize}(\tau) & \text{if } p(\tau) \geq x - x^2 \\ \text{monetize}(\tau) & \text{otherwise} \end{cases}
\end{aligned}
$$

Outline

Static incompleteness and dynamic completeness

World of states

String diagrams

Universal language

Self-confirming beliefs

Self-updating explanations

Training and constructing bot beliefs

Empiric problem in machine learning

Why do bots hallucinate?

Foundational problem in belief logic

How do false beliefs evolve:

- myths, legends, religions
- lies, deceptions
- delusions

?

Foundational problem in belief logic

How do false beliefs evolve:

- myths, legends, religions
- lies, deceptions
- delusions

?

How do they survive the reality checks?

General approaches

- induction $\rightsquigarrow$ formal epistemology
  - Bayes, Fisher, Carnap, Popper, Jaynes, Dempster-Shafer, Vapnik, ...
- incomplete information $\rightsquigarrow$ interactive epistemology
  - Harsanyi, Aumann, Brandenburger, DEL (Baltag, van Benthem, van Ditmarsch, ...)

Insight

Belief engineering (campaigning) combines

- belief logic
- algorithm design (programming)
- universal (programming) languages.

Insight

Belief engineering (campaigning) combines

- belief logic
- algorithm design (programming)
- universal (programming) languages.

Universality allows

- unfalsifiable claims
- dynamic completeness.

Tentative answers

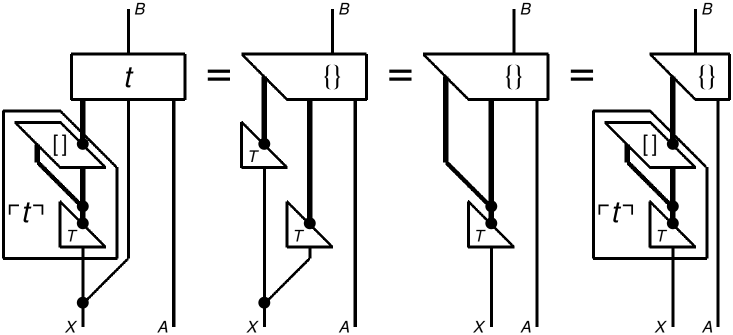
Bots hallucinations arise through

- *data processing*:
  dreaming without sleeping
- **logical constructions**:
  self-confirming explanations

False beliefs persist through

- *data processing*:
  ~~concept amplification~~
- **logical constructions**:
  self-updating (unfalsifiable complete) explanations

“Credo quia absurdum”



"Your friends are my followers"