Subsumption-Linear Q-Resolution for QBF*

- What is Subsumption-Linear Q-Resolution (SLQR)?
- The paper's main result about SLQR
- Why SLQR is of interest:
  Search Complexity vs. Proof Complexity
- Main Proof Idea of Anderson and Bledsoe (1970)
- A Word about QBF Models and the Two-Player Game
- Questions

*Full paper:https://users.soe.ucsc.edu/~avg/Papers/slqr-long.pdf

Search Complexity vs. Proof Complexity

- Prof. Toniann Pitassi at Univ. of Toronto and co-authors pioneered study of Proof **Search** Complexity:
- Even though a formula family has “short” refutations, does it take exponential time for a proof system to **find** one?
- The motivation for (propositional) SLR was efficient **proof search**, rather than short refutations.
- SLQR is similarly motivated.

What is SLQR?

- **Linear** means *first* clause for next resolution operation **must be** the most recently derived clause.
- The second clause may be any *input* clause or an *earlier derived* clause.
- Old name: **Selection** Linear Resolution
- **Selection** means *any* literal in the first clause of a resolution operation may be selected as the *clashing literal*.
- If no refutation can be found, backtracking on **choice of clashing literal** is unnecessary for completeness.
- Backtracking on the **choice of second clause** is needed for completeness.
- Many clauses may qualify as the second clause, **including earlier-derived clauses.**

Subsumption Linear Resolution: later papers

- **Subsumption Resolution** occurs when the resolvent **properly** subsumes one of the operands.
- E. g., Res($[\neg e, p, q, r], [e, q]$) $= [p, q, r]$.
- The main results for propositional resolution and first-order resolution are that:
  An earlier-derived clause needs to be considered as the second clause of a resolution operation
  ONLY IF that clause permits subsumption resolution.
- This paper extends that property to **Quantified Boolean Formulas**.

Anderson-Bledsoe (1970) Completeness Idea

- **Restriction on** v for a quantifier-free CNF formula F means "assign v to 1 (true) and apply truth-value simplifications".
- **Restriction on** $\neg v$ for a quantifier-free CNF formula F means "assign v to 0 (false) and apply truth-value simplifications".
- Restrict F on $v = 1$ giving $F1$ and refute $F1$ giving $R1$.
- Restrict F on $v = 0$ giving $F0$ and refute $F0$ giving $R0$. W.L.O.G. The top clause of $R0$ may be an original clause that contained v.
- Put back $\neg v$ in $R1$, deriving unit clause $[\neg v]$. Call this derivation $R1B$.

The AB70 Idea Applies to SLR (cont'd)

- **Concatenate the derivations** $R1B$ and $R0$, **except** whenever a clause with v is derived, immediately insert resolution with $[\neg v]$.
- The last clause thus derived is the empty clause.
- The **Subsumption Linear** restriction is satisfied because resolution with a unit clause is **subsumption resolution**.

QBF Models and the Two-Player Game

- A **QBF Model** is a **set of Herbrand functions**, $\{H_e\}$, one of each **existential** variable e.
- H_e depends on **universal** variables outer to e in the prenex. (If e is outermost in the prenex, H_e is a constant 0 or 1.)
- The **A** player tries to make the QBF **false** by choosing a value for the outermost unassigned universal variable.
- The **E** player tries to make the QBF **true** by choosing a value for the outermost unassigned existential variable.
- Choices continue until the **matrix** (quantifier-free part) simplifies to 1 or 0.
- If the QBF has a model, the **E** player wins by choosing according to H_e.