Cosmology

Schutz – GR – Ch. 12.
Weinberg – physical cosmology
Ellis – Varenna – math cosmology.

articles: Ellis
MacCallum.

Large Scale Structure

galaxies | clusters small scale structure

Physics – Grav. fileld

general relativity

Homogeneity:

observed large scale uniformity

uniformity on scales 10^3 Mpc

- density
- galaxies / clusters
- element abundances

Isotropy:

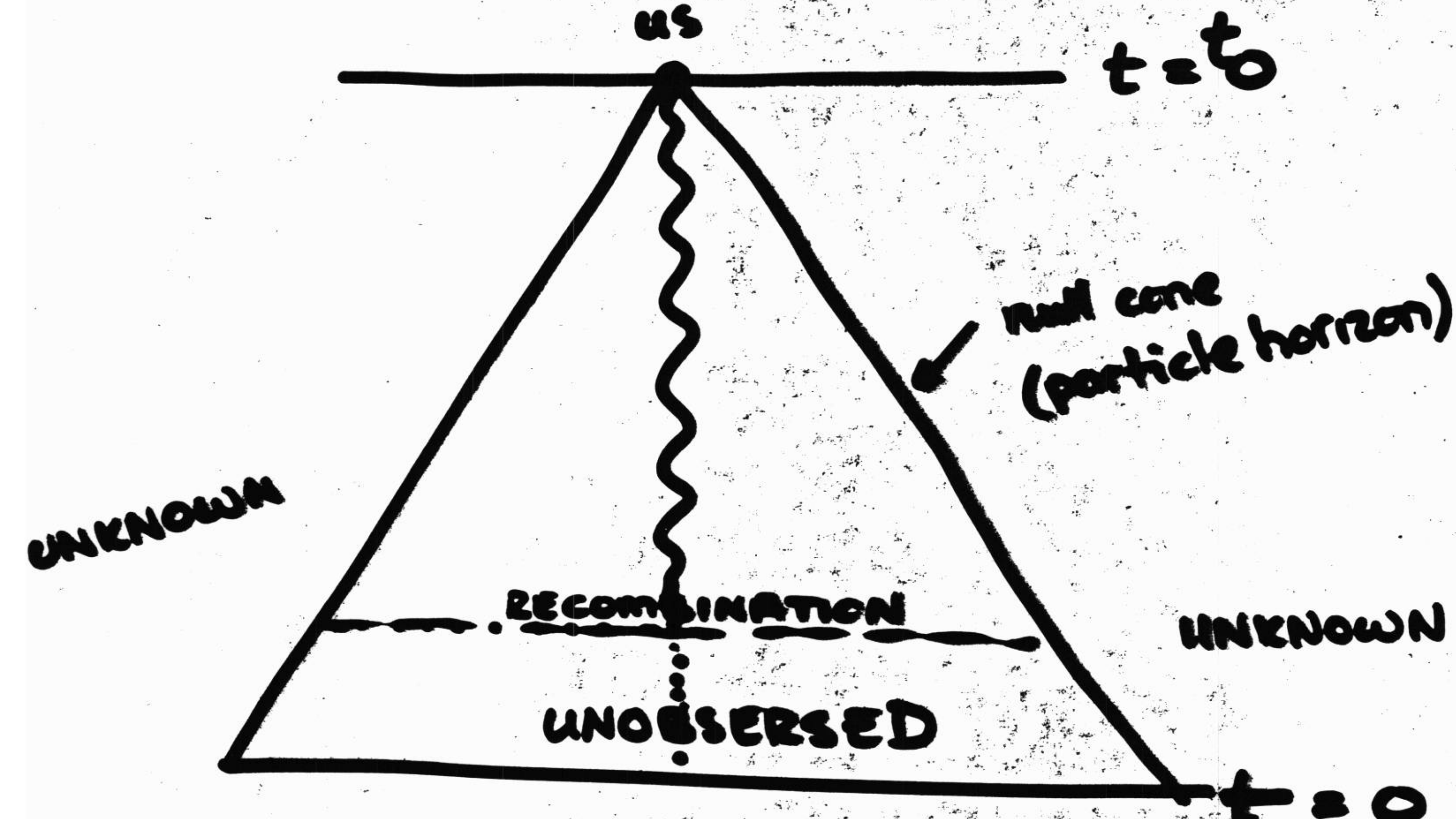
universe isotropic (same in all directions)

- number counts
- CMB
- Hubble expansion

Hubble expansion

all galaxies appear (on average) to be receding from us (speed $\propto$ dist.)

Finite universe



see along null cone (astron obs)
own world line (geological data)

Cosmological Models

Assumptions

1. Cosmological Principle.
   Universe everywhere SH and I

2. Perfect Fluid source.
   μ, P, u^a – comoving velocity
   ('galaxies' comoving with fluid)

3. Gravitational field.

4. GR.

METRIC

$$SH + I \Rightarrow (p322\text{-}5)$$

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{12.14}$$

Friedmann-Robertson-Walker models.

R ~ scale factor

$$k \text{ (normalized)} = \begin{cases} +1 & \text{CLOSED} \\ 0 & \text{FLAT EUCLIDEAN} \\ -1 & \text{OPEN} \end{cases}$$

$$^3R_0 = \begin{cases} \text{metric of three sphere radius } R(t_0) \\ \text{zero} \\ \text{constant negative curvature (hyperbolic} \end{cases}$$

EFE's $\mu(t),\ p(t),\ u^a = +\delta^a_0$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{-k}{R^2} + \mu \qquad (12.27)$$

$$\left[\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -3p\right]$$

Conservation eq^n $\left(T^{\mu\nu}{}_{;\nu} = 0\right)$

$$\frac{d}{dt}\left[\mu R^3\right] = -p\frac{d}{dt}\left[R^3\right] \qquad (12.21)$$

Matter dominated epoch

(present epoch)

$$P_{(m)} = 0 \quad ; \quad \mu_{(m)} = c R^{-3}$$

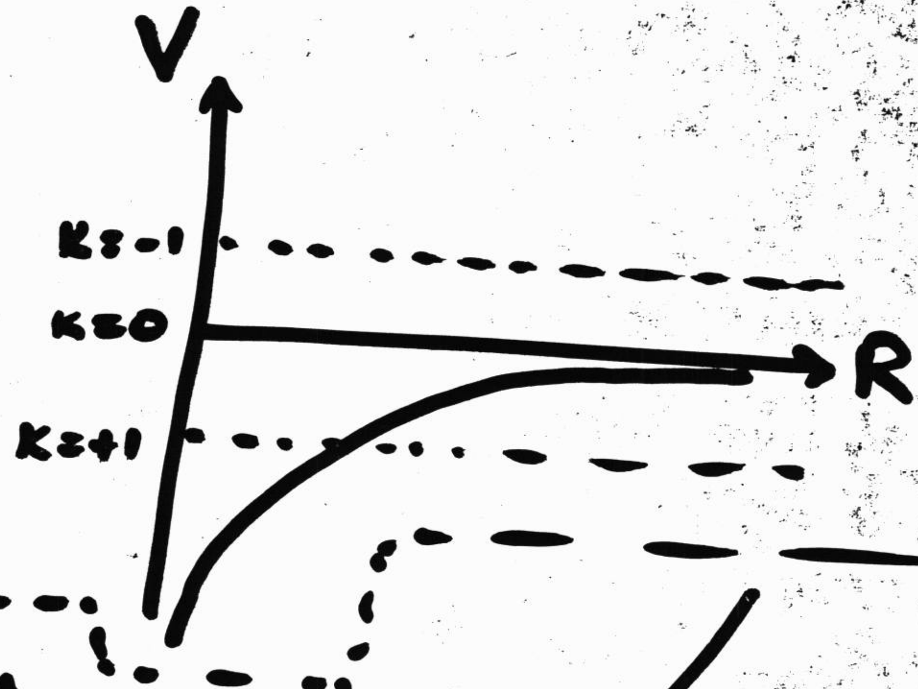
Radiation epoch

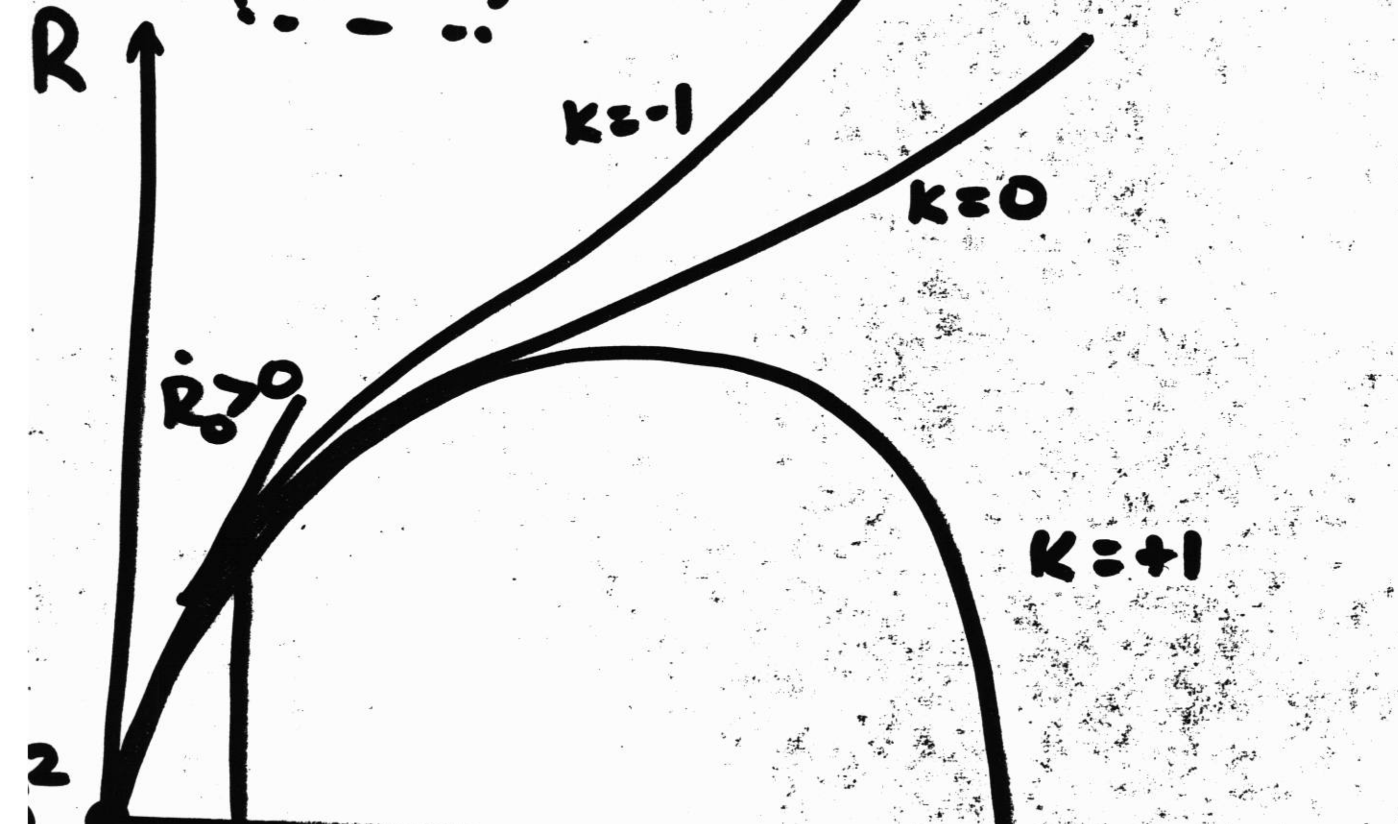
(early universe)

$$P_{(r)} = \frac{1}{3} \mu_{(r)} \quad ; \quad \mu_{(r)} = c R^{-4}$$

Matter universe

$$\dot{R}^2 = \frac{c}{R} - k \equiv -V - k$$





All models have Big Bang (B^2) singularity

$K=0,-1$ expand forever

$K=+1$ recollapses.

Radiation universe

Early epochs

$$\dot{R}^2 = \frac{c}{R^2} - K$$

NO QUALITATIVE DIFFERENCE.

For R small, curvature term negligible (i.e. flat model adequately describes dynamics)

Scale t st $R(0) = 0$ (singularity occurs at $t=0$)

Solution

$$\boxed{R(t) = (4c)^{1/4}\, t^{1/2}}$$

Note $\mu_m \propto R^{-3}$
$\mu_r \propto R^{-4}$

Thus radiation dominant early time (R small), matter dominant late times.

At t_e : $\mu_m = \mu_r$

$t < t_e \quad \mu \simeq \mu_r$ (radn era)
$t > t_e \quad \mu \simeq \mu_m$ (matter era)

To get model $\forall t$ 'match' radiation model to matter model at $t = t_e$.

NOTE: $t_e \simeq$ time recombination ($T_r \sim 4000$

Big Bang singularity

as $t \to 0^+$

- $R(t) \to 0^+$
- $\mu(t) \to \infty$
- $T_r(t) \to \infty$

singularity "point-like".

When does this description fail?

- singularity generic? (Yes: singularity theorems – Hawking).
- nature singularity in general? (relax I or SH).

Cosmological Observations

(ELLIS)

- Hubble parameter $H_0 = \left.\frac{\dot{R}}{R}\right|_0$

Need accurate distance scale d (luminosity distance)

(i) closer: d more accurate – but cosmological redshift harder to dissociate

(ii) further: d less accurate:

$$\boxed{50 \leq H_0 \leq 95 \ \text{Km s}^{-1}\text{Mpc}^{-1}} \text{ (error in } d\text{)}.$$

- Deceleration parameter $q_0 = -\left.\frac{R\ddot{R}}{\dot{R}^2}\right|_0$

measure distant objects – see them as they were when light emitted (when $H(t)$ had different value to H_0); thus measure rate of change of H (i.e q_0)

Difficult: need to know how galaxies evolve (Luminosity ⇒ lum. dist.)

- density of universe

$\mu_c = 3H^2$ in flat universe

- depends on $H(t)$!

$\mu_{c,0} = 3H_0^2$

density parameter Ω

$$\Omega = \frac{\mu_0}{\mu_{c,0}} = 2q_0$$

Thus $\Omega > 1$ closed universe

$\Omega < 1$ open universe

$\Omega_{lum} \sim .01$ (obs)

$\Omega_{rot} \sim .2$ (obs) DM?

$\Omega_{nuc} \lesssim .2$ (obs/th)

$\Omega_{inf} \cong 1$ (th.) DM?

- element abundances.
- CMB measurements.
- Large scale structure.

Theoretical Considerations:

- particle physics
- galaxy formation
- Quantum gravity
- inflation
- cosmic strings

Physical Cosmology

(WEINBERG)

History: $t \to 0^+$ or $T \to \infty$

- present matter epoch

  galaxy and star formation
  (theory of density perturbations).

- recombination $T \sim 4000\,K$

  recombination of ionized plasma into neutral hydrogen

  $T < 4000$ — hydrogen and freely prop. photons (CMB)

- radiation era $T > 4000K$
  - $\mu_r > \mu_m$
  - Themal eq^m $\rightarrow T_r = T_m$

- Primordial Nucleosynthesis

$$T \sim 5\times10^9 - 10^{11} K$$
$$(t \sim 1 - 200 s)$$

temperatures at which nuclear reactions (similar to those in stars) can take place.

$T > 10^{11} K$ – thermal energies break up nuclei

- protons (H nuclei), neutrons, electrons neutrinos, photons, other particles.

As T decreases

- weak reactions cease; n, p in eq^m abundance
  neutrinos decouple
- nuclear reactions begin

- neutrons freeze out
- H_2, D, ^4He, ^4He, other light elements produced.
- using: $R(t) \propto t^{1/2}$
  nuclear reaction rates
  present density (baryons)
  number N_γ
  neutron half life

Can calculate primordial abundances
(Computer: Wagoner)

Consistent observations

- Earlier times

(Particle physics)

  - Lepton era (muons)
  - inflation ?
  - hadron era (quarks)
  - GUTs
  - Quantum Gravity