INFLATION

critique: Ellis

Standard model

SUCCESSES

- explain Hubble $z - r$ relation in terms expansion universe; H_0^{-1} consistent with ages for stars and galaxies
- explain CMB and nucleosynthesis of light elements in terms of hot early phase universe

PROBLEMS?

- why is universe so smooth? (cf $|\Delta T/T| < 10^{-5}$ for CMB).
- what is origin of structures we see?
- horizon problem: i.e. limits on causal effects in early universe due to particle horizons.
- Ω: only know to within two orders of magnitude (final evolution?). Flatness problem

INFLATION

- introduces scalar field ϕ which dominates at early times but eventually decays away - its energy being converted to radiation
- various models ('old', 'new', 'chaotic', ['extended']) depending on particular particle microphysics.
- ϕ field gives rise to 'effective' cosmological constant that drives universe through period of exponential expansion.
- after reheating - standard scenario follows (baryosynthesis, nucleosynthesis, etc)

SUCCESSES

- solves horizon problem
- solves smoothness | isotropy problem

- both create and solve 'monopole' problem

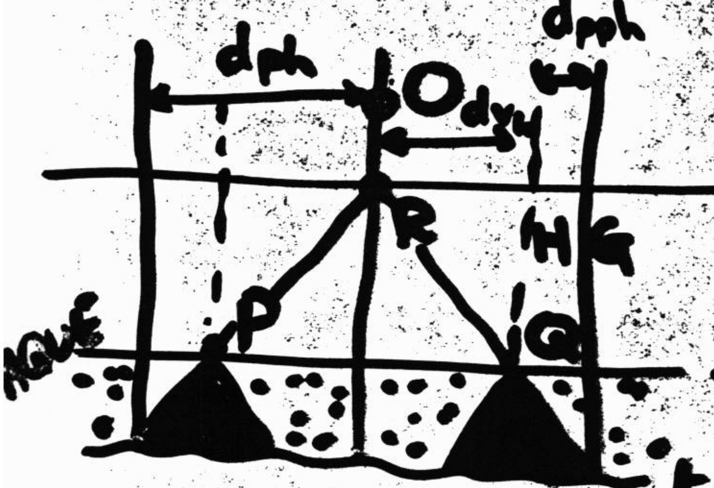
- almost solve structure problem - Inflation predicts scale - free spectrum of density perturbations:
  - but amplitude wrong ?
  - evidence spectrum not scale-free

- solve flatness problem ?
  - too well - inflation predicts $\Omega \sim 1$
  - but observations indicate $\Omega < 0.1$ - prediction of dark matter to reconcile theory.

HORIZON PROBLEM

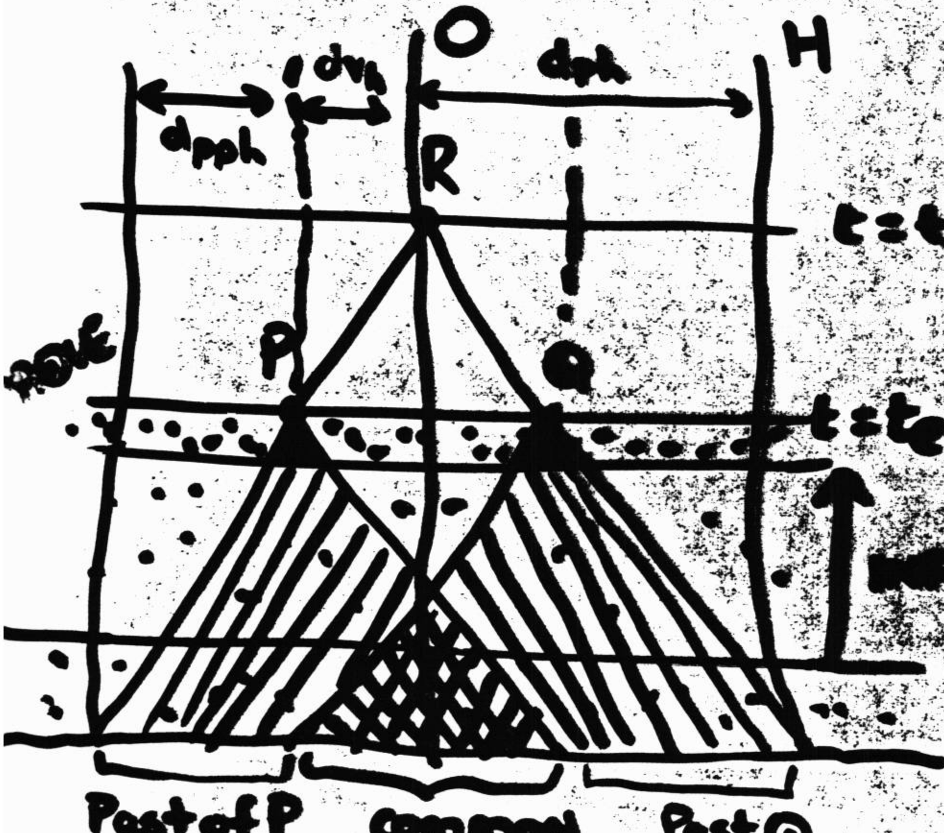
(use conformal coords st
$\eta = \int_0^t dt/S(t)$:
light rays at $\pm 45°$)

Standard Big Bang



$\boxed{d_{vh} > d_{pph}}$

Inflation



$\boxed{d_{pph} > d_{vh}}$

FAILURES ?

- identity 'inflaton' not known (i.e. inflation is idea rather than specific physical theory).
- no actual proof inflation has taken place.
- role and constitution of dark matter.

<u>Theory</u> ϕ - inflaton
$V(\phi)$ - potential

$$(*) \quad T^{\phi}_{ab} = \phi_{;a}\phi_{;b} - g_{ab}\left(\tfrac{1}{2}\phi_{;c}\phi^{;c} + V\right)$$

If $\phi \sim \phi_0$ — $V_0(\phi) = V(\phi_0) \sim$ const

$$T_{ab} = -g_{ab}V_0 \longrightarrow \text{cosm. const.}$$

Take $\phi_{;c}\phi^{;c} < 0$ (true in FRW models) st ϕ = const SL surfaces : choose u^a normal to these surfaces (i.e u^a is unique TL eigenvector of Ricci tensor) — then

(*) takes perfect fluid form with

$$\mu_\phi = \tfrac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \tfrac{1}{2}\dot{\phi}^2 - V(\phi).$$

with

$$T_{ab}^{tot} = T_{ab}^{matter} + T_{ab}^{\phi}$$

(provided all u^a same): still perfect fluid with

$$\mu = \tfrac{1}{2}\dot{\phi}^2 + V(\phi) + \mu_r + \mu_m$$

$$p = \tfrac{1}{2}\dot{\phi}^2 - V(\phi) + \tfrac{1}{3}\mu_r$$

where $\mu^{matter} = \mu_r + \mu_m$
and $p_m = 0$ and $p_r = \frac{1}{3}\mu_r$

Non interaction: separate conservation

$$\ddot{\phi} + \Theta\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$\dot{\mu}_m + \mu_m\Theta = 0$$

$$\dot{\mu}_r + \tfrac{4}{3}\mu_r\Theta = 0$$

write $p = (\gamma - 1)\mu$: $\gamma = \dfrac{\dot{\phi}^2 + \frac{4}{3}\mu_r + \mu_m}{\frac{1}{2}\dot{\phi}^2 + V + \mu_r + \mu_m}$

$$\Omega \equiv \frac{\mu}{3H^2} = \left(\tfrac{1}{2}\dot{\phi}^2 + V(\phi) + \mu_r + \mu_m\right)/3H^2$$

FRW models: $\begin{cases} \Theta = \dfrac{3\dot{R}}{R} = 3H \\ \mu, p \text{ arbitary} \end{cases}$

$$3\dot{H} + 3H^2 = \tfrac{1}{2}(\mu + 3p) \qquad \text{(Raych.}$$

$$3H^2 + 3K = \mu \qquad \text{(Friedm}$$

where $K = \dfrac{k}{S^2}$; k const (spatial curv)

(Total) energy conserv^n:

$$\dot{\mu} + 3H(\mu + p) = 0$$

or

$$\dot{\mu} + 3H\gamma\mu = 0$$

From these relations:

$$\Omega = 1 + \frac{k}{S^2 H^2}$$ (Nb $\Omega = 1$ critical value)

Defining deceleration parameter $q \equiv -\ddot{S}/SH^2$

$$q = \left(\tfrac{3}{2}\gamma - 1\right)\Omega$$ ($\gamma = 2/3$ is critical value)

From above obtain "phase plane eqⁿ"

$$\frac{d\Omega}{dS} = (2-3\gamma)\Omega(1-\Omega)/S$$

showing how Ω evolves (for given γ).

Notes: 1. $\Omega = 1$ and $\Omega = 0$ solⁿs eqⁿ.

2. $\Omega > 0$ any time: $\Omega > 0 \ \forall t$

3. $0 < \Omega < 1$ any time: true $\forall t$.

At present time t_0:

$$\mu_0 = 3\Omega_0 H_0^2 \ : \ S_0 = \frac{1}{H_0\sqrt{|1-\Omega_0|}} \quad k^2 = 1$$

S_0 arbitrary $\quad k = 0$

Inflation

$\gamma = \text{const} < 2/3 \rightarrow q < 0 \rightarrow$ accelerating

- called <u>inflationary</u> model

(a) $\gamma = 0$ — exponential
$\gamma \neq 0$ — power law

(b) usual energy conditions violated.

if ($t_i \sim$ time inflation)

$\Omega = 1$ at t_i : $\Omega = 1 \; \forall t$

$\Omega > 1$ at t_i : $\Omega \rightarrow 1^+$ t increases t_f

$\Omega < 1$ at t_i : $\Omega \rightarrow 1^-$ $t \rightarrow t_f$

(t_f: time end inflation)

i.e. inflation drives Ω towards unity

Inflationary Model

1. t_p to t_i : preinflationary era : μ_ϕ, μ_r

2. t_i to t_f : inflationary era : μ_ϕ, μ_r

3. t_f : reheating : $\mu_\phi \rightarrow \mu_r$
(such a way not to cause inhomogeneities)

4 t_f onward : post-inflationary phase : μ_r, μ_m

[γ, (S and μ) cont^s at changes]
($\rightarrow$ curves in (Ω, S) plane cont^s there)

Details (expanding universe)

$\Omega = 1$: evolves from $S \rightarrow 0$ to $S \rightarrow \infty$ with $\Omega = 1$

$\Omega < 1$: Ω decreases from 1 (lim as $S \rightarrow 0$) to $S = S_i$, increasing to max at $S = S_f$, finally decreasin to 0 as $S \rightarrow \infty$

$\Omega > 1$: starts from $\Omega \to 1$

(a) Ω increases to max at $S = S_i$, decreases to min. at S_f, increases to infinity at max. expansion, then undergoes (time-symmetric ?) collapse with $\Omega \to 1^+$ at final crunch.

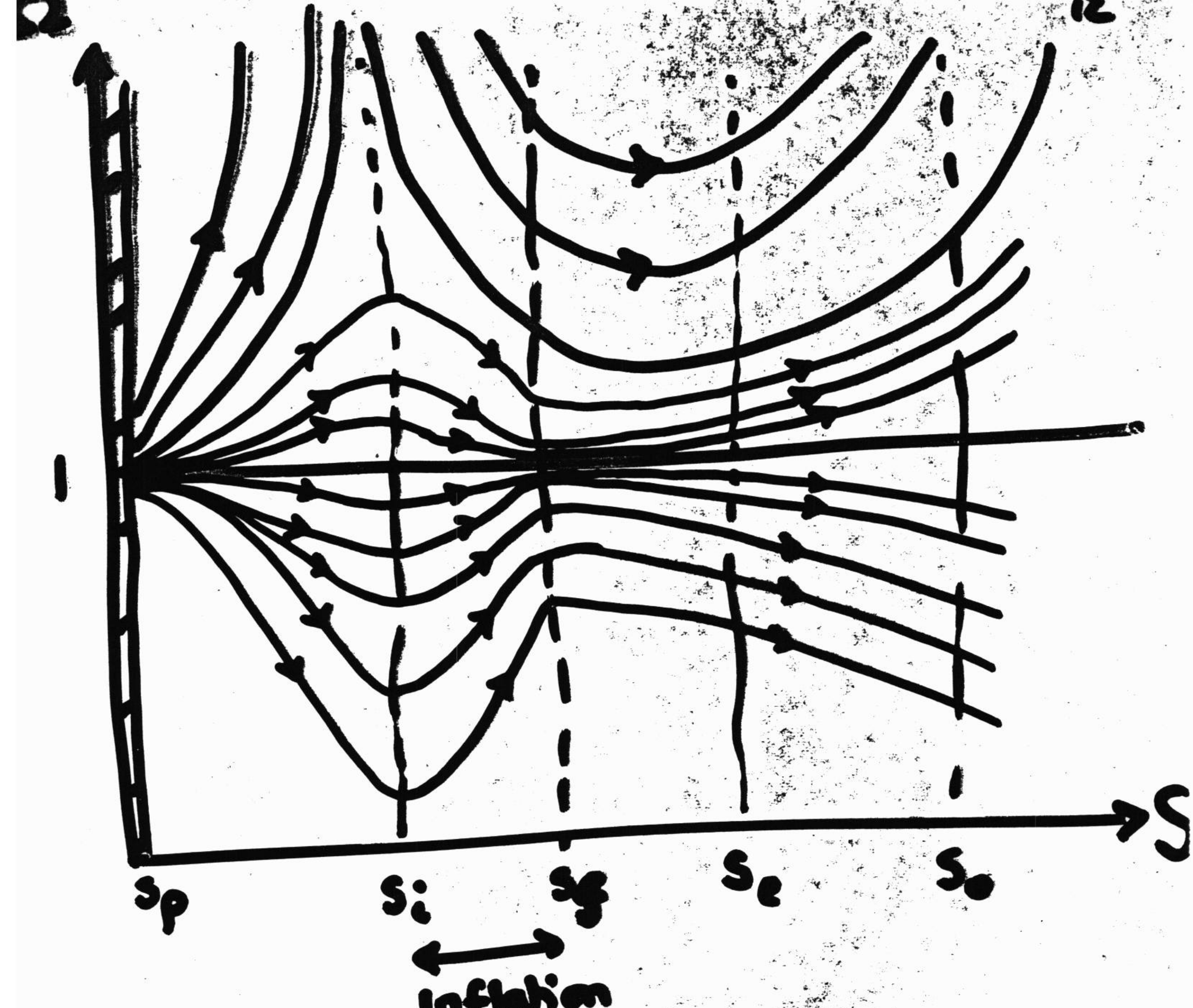
(b) universes that recollapse before onset of inflation: $\Omega \to \infty$ (at max exp.) and $\Omega \to 1$ as universe recollapse.

(c) separatrix of two behaviours: model asymptotically approaching Einstein static universe E.

(d) Two other separatrices:

(i) models start off at E, expand to max before collapsing to E

(ii) models start off from E and collapse to singularity in future.



Generic evolution (i.e no particular mechanism for inflation)

Commentary

- inflation drives Ω toward 1 at S_f: $\Omega \sim 1$ at S_e ($\Omega \sim 1$ at S_0).

- however, $\exists$ integral curve thru each pt and hence there are models (initial cond^{n}s) with Ω not close to 1 (inflation in itself does not solve 'flatness problem'.) [i.e inflation compatible with $\Omega_0 = 0.2$]

- however, $|\Omega - 1| \leq 1$ today implies $|\Omega - 1| \ll 1$ at decoupling and earlier.

- maybe inflationary models exist with $\Omega_0 = 0.2$ but they are not probable: only $\Omega \simeq 1$ universes probably. To answer this question we need a time-depend measure

- fix initial conditions and t_0: vary physics — can make Ω_0 as as close to 1 as desired
  But not legitimate approach

- fix initial conditions and physics: vary t_0:
  for $\Omega > 1$: can measure any value Ω_0 (dep t_0)
  for $\Omega < 1$: for most t_0 $\Omega_0 \simeq 0$.

  $\Omega \simeq 1$ only if we restrict t_0 to relatively small epoch after t_f in those universe in which Ω_f close to 1

  Anthropic principle ?

- fix physics and t_0: vary initial conditions.
  i.e choose value Ω_p (probability: need measure on initial data).

  most likely $\Omega_p \gg 1$ (if all values Ω_p allowed: $0 < \Omega_p < \infty$) ?
  But in this case universe will recollapse before t_0!

<u>objection</u>

Fluid approximation used : proper scalar field dynamics

FRW geometry : drop matter term.
$\phi = \phi(t)$.

$$H^2 + K = \frac{1}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$
$$\dot{H} + H^2 = \frac{1}{3}\left[V(\phi) - \dot{\phi}^2\right]$$
$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$
$$\dot{\mu}_\phi + 3H\dot{\phi}^2 = 0.$$

(only 2 indep^t eq^{n}s).
main inflationary era: slow rolling approx $\ddot{\phi} \ll 3H$
end of inflation / approach reheating : fast rolling app

To do physics need $V(\phi)$.
appeal to particle physics for form $V(\phi)$.
But $\exists$ $V(\phi)$ leading to any desired $S(t)$!

density pert's: $\delta\mu/\mu$

- inflationary expansion smooths out universe until it is like FRW (explaining CMB): any inhomogeneities are 'pushed outside' visual horizon

- quantum fluctuations (of scalar field) 'within horizon' (i.e. on scales less than expansion scale length) give rise to new perturbations on small scales

C: quantum fluctuations form galaxy seeds – triumph of theory that it is able to produce causal mechanism which has potential to explain LS structure

- size of fluctuations increase enormously during de Sitter expansion: since H is constant then (sol^n. time transl^n invar^t) – scale-free spectrum of pert^{n}s

C: unique prediction scale-free spectrum observations?

C: Double inflation

C: non-Gaussian, non-scale-free pert^{n}s possible in power law inflation.

- during (exp^l) expansion small scales grow until they exceed expansion scale (which is constant): 'cross horizon'

Hubble scale $\lambda_H = H^{-1}$: $\mathcal{H} = \lambda/\lambda_H$ –

$\mathcal{H} > 1$ 'outside horizon'
$\mathcal{H} < 1$ 'inside horizon'.

- perturbations grow 'outside horizon'.

- inflation ends, scalar field converted radiation. Hubble scale increases, successively exceeding larger and larger length scales (correspondin pert^{n}s 're-enter horizon'). Pert^{n}s on these scales oscillate or start to grow providing seeds for growth galaxies and large scale structure.

Summary:

- only restricted initial data will lead to observed universe.

- inflation is good at breaking horizons and smoothing out universe, so explaining CMB isotropy.

- promising in terms of structure generation

- does not force $\Omega \sim 1$ at present time.

Probability $\Omega \sim 1$ open question.