MATH 2300 Assignment 3

This assignment is due Tuesday March 29, 2016 at the beginning of class. **No unstapled assignments will be accepted.**

1. Fit a low degree polynomial to the points

```
> p:=[[0,0],[1,2],[2,4],[3,6],[4,8],[5,10.001],[6,12],[7,14]];
```

2. Write a Maple procedure **atMostQuarticFit** that takes as input a list of points *p* and a positive integer i between 1 and 4 inclusive, and returns a best fitting polynomial of degree *i* to *p*. (You willl have tofind a way to find the best fitting quartic.

3. Consider the points

```
> p:=[[20,20],[25,28],[30,40.5],[35,52.5],[40,72],[45,92.5],[50,118],
  [55,148.5],[60,182],[65,220.5],[70,266],[75,318],[80,376]];
```

(a) Best fit a straight line to p.

(b) Best fit a quadratic to p.

(c) Best fit a cubic to p.

(d) Best fit a quartic to p.

(e) Which model best fits the trend of the data p? Don't necessarily choose the highest degree polynomial. Explain your answer (you are graded on your reasoning).

4. Use a divided difference table to decide whether a low order polynomial fit the following points:

```
> p := [[0, -0.02], [1, 11.879], [2, 41.984], [3, 114.871], [4,
  255.196], [5, 487.455], [6, 836.416], [7, 1326.431], [8, 1982.444],
  [9, 2828.599], [10, 3890.080]];
```

5. Write a Monte Carlo Maple procedure to find the area below a curve f, above a curve g, between x = a and x = b. Check your procedure on $f(x) = x^3$, $g(x) = -x^2 - 1$, between x = 0 and x = 3.

6. Write a Monte Carlo Maple procedure to estimate the volume trapped between the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + 3\,y^2$. Explain any mathematics that you need to use (e.g. solving equations) for the simulation. Also, run your simulation and compare its output values to the actual volume.

7.

The distribution of some values are normally distributed with mean mu and standard deviation sigma if, for any number x, the probability that a value is less than x is $\int_{-infinity}^{x} \frac{e^{-\frac{(t-\mathrm{mu})^2}{2\,\mathrm{sigma}^2}}}{\mathrm{sigma}\,\sqrt{2\,\mathrm{Pi}}}\,dt$.

(Here mu is any real number and sigma is any positive number).

(a) Plot the integrand with mu = 0 and sigma = 1 for x = -10 to 10.

(b) Why is the integrand always positive?

(c) Set mu = 0 and sigma = 1. Use a Monte Carlo simulation to complete the following table: **(1)**

		B	C	D	E
1	*x*	*probability*			
2	−3				
3	−2.5				
4	−2				
5	−1.5				
6	−1				
7	−0.5				
8	0				
9	0.5				
10	1				
11	1.5				
12	2				
13	2.5				
14	3				

8. Suppose we have the following matrix for a Markov chain with 5 states:

```
> with(linalg):
  m:=matrix([[1/5,1/5,1/5,1/5,1/5],[1/10,1/10,2/5,1/5,1/5],[2/5,1/5,
  1/5,1/10,1/10],[1/5,2/5,1/10,1/5,1/10],[1/5,1/5,1/5,3/10,1/10]]);
```

(2)

$$m := \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{10} & \frac{1}{10} & \frac{2}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{10} & \frac{1}{10} \\ \frac{1}{5} & \frac{2}{5} & \frac{1}{10} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{10} & \frac{1}{10} \end{bmatrix} \quad \textbf{(2)}$$

(a) For the initial state

```
> s:=matrix([[0,1,0,0,0]]);
```

$$s := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \textbf{(3)}$$

What is the probability you are in state 4 after 4 moves?

(b) What is the steady state matrix?

9. Suppose a markov chain has a transition matrix with a row or a column of 0's. Prove that the markov chain is <u>not</u> regular.