```
#
# Best Fiiting a line of the form y = Ax+B to data points
# (x_1,y_1),...,(x_m,y_m)
#


bestFitLineThroughOrigin := proc(pts)
 local n, dn, nm, i, C :
 n := nops(pts) :
 dn := 0 :
 nm := 0 :
 for i from 1 to n do
 nm := nm + pts[i][2]·pts[i][1] :
 dn := dn + (pts[i][1])^2 :
 od:
 C := nm/dn :
 x→C·x;
 end
```

```
proc(pts)                                                                    (1)
    local n, dn, nm, i, C;
    n := nops(pts);
    dn := 0;
    nm := 0;
    for i to n do nm := nm + pts[i][2]*pts[i][1]; dn := dn + pts[i][1]^2 end do;
    C := nm/dn;
    x→C*x
end proc
```

```
bestFitLine := proc(pts)
 local m, E, F, G, H, i, x, y, A, B :
 m := nops(pts) :
 E := 0 :
 F := 0 :
 G := 0 :
 H := 0 :
 for i from 1 to m do
  x := pts[i][1] :
  y := pts[i][2] :
  E := E + x^2 :
  F := F + x :
  G := G + x·y :
  H := H + y :
 od:
 A := (F·H - m·G)/(F^2 - m·E) :
```

$$B := \frac{F \cdot G - E \cdot H}{F^2 - m \cdot E};$$

$x \rightarrow A \cdot x + B;$

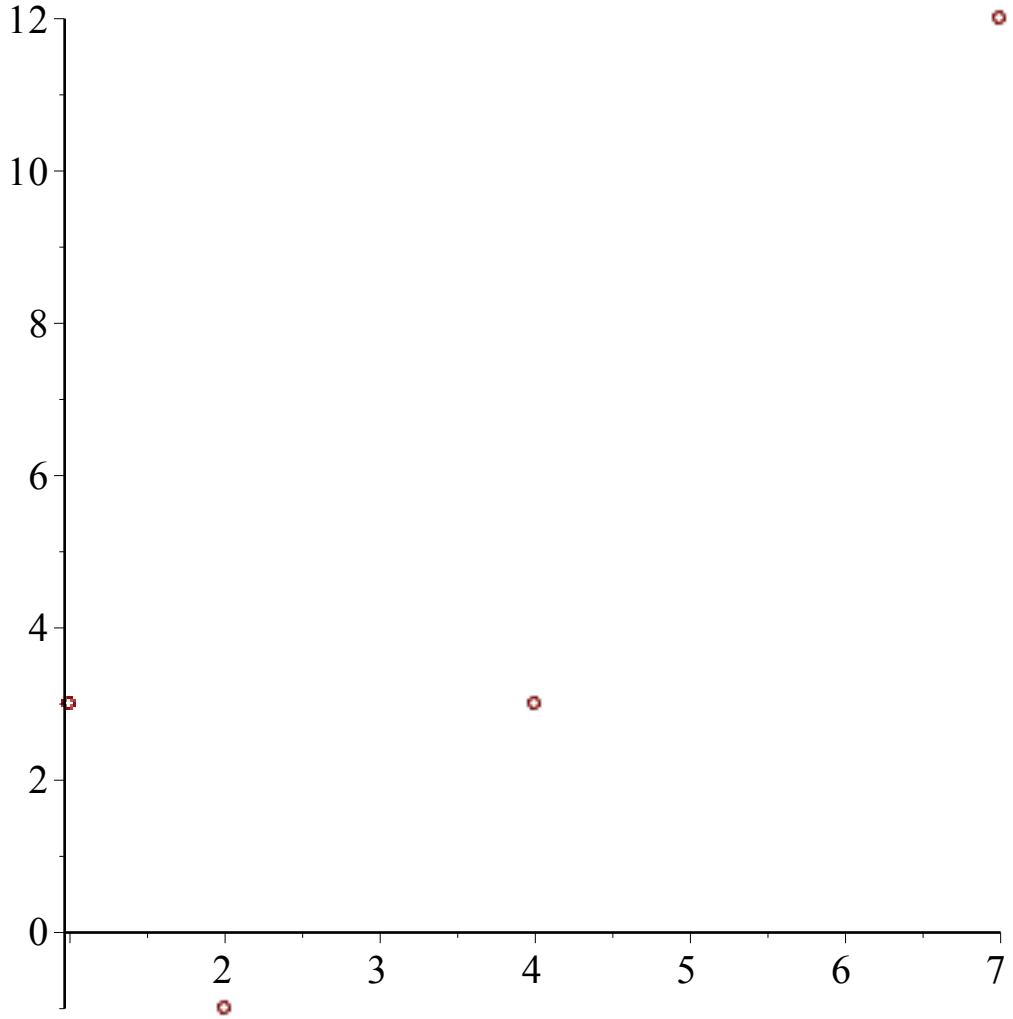
end

```
proc(pts)                                                                    (2)
    local m, E, F, G, H, i, x, y, A, B;
    m := nops(pts);
    E := 0;
    F := 0;
    G := 0;
    H := 0;
    for i to m do
        x := pts[i][1]; y := pts[i][2]; E := E + x^2; F := F + x; G := G + x*y; H := H + y
    end do;
    A := (F*H - m*G)/(F^2 - m*E);
    B := (F*G - E*H)/(F^2 - m*E);
    x -> A*x + B
end proc
```

```
pts := [[1, 3], [2, -1], [4, 3], [7, 12]];
                                [[1, 3], [2, -1], [4, 3], [7, 12]]           (3)
```

```
with(plots);
[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d,   (4)
    conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot,
    display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d,
    inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d,
    listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto,
    plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d,
    polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions,
    setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d,
    tubeplot]
```

```
plot1 := plot(pts, symbol = circle, style = point) :
display(plot1);
```



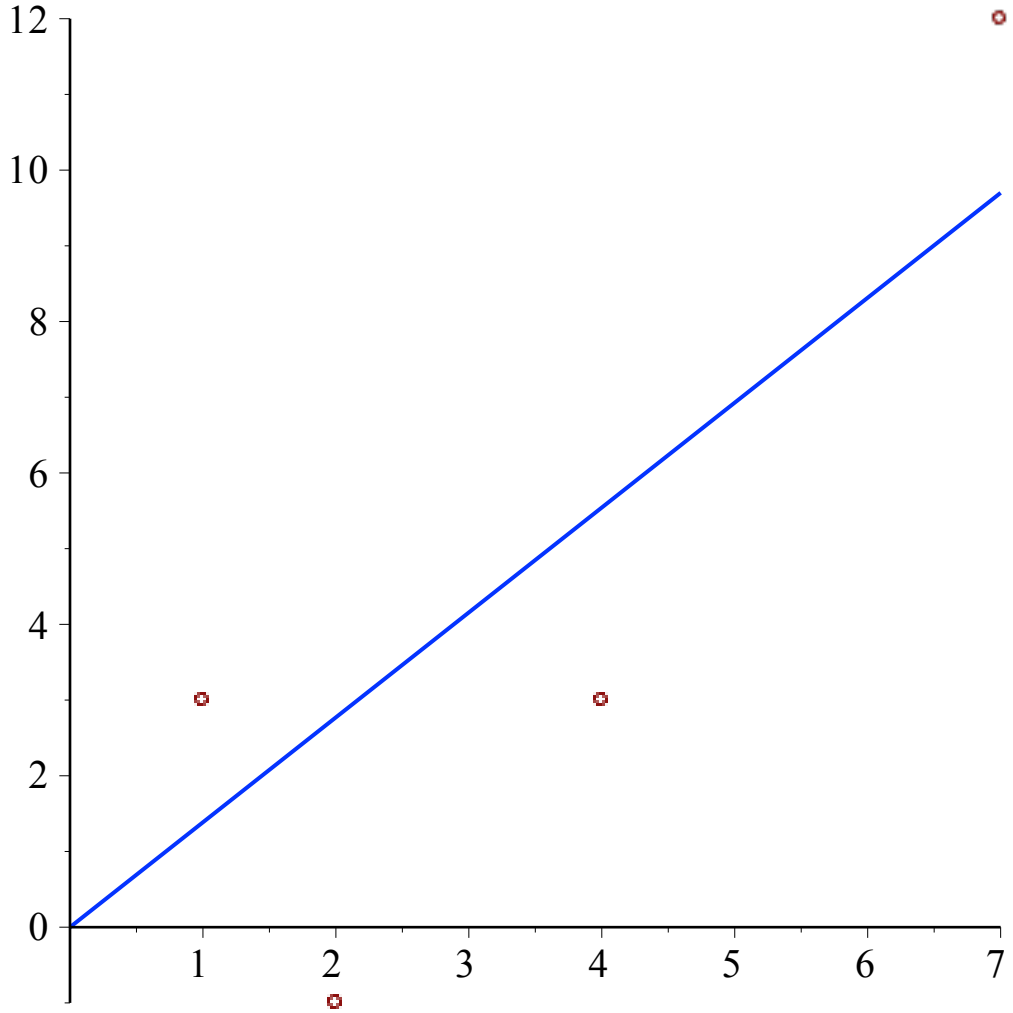
$f := bestFitLineThroughOrigin(pts);$
$g := bestFitLine(pts);$

$$x \rightarrow C\,x$$
$$x \rightarrow A\,x + B \qquad \textbf{(5)}$$

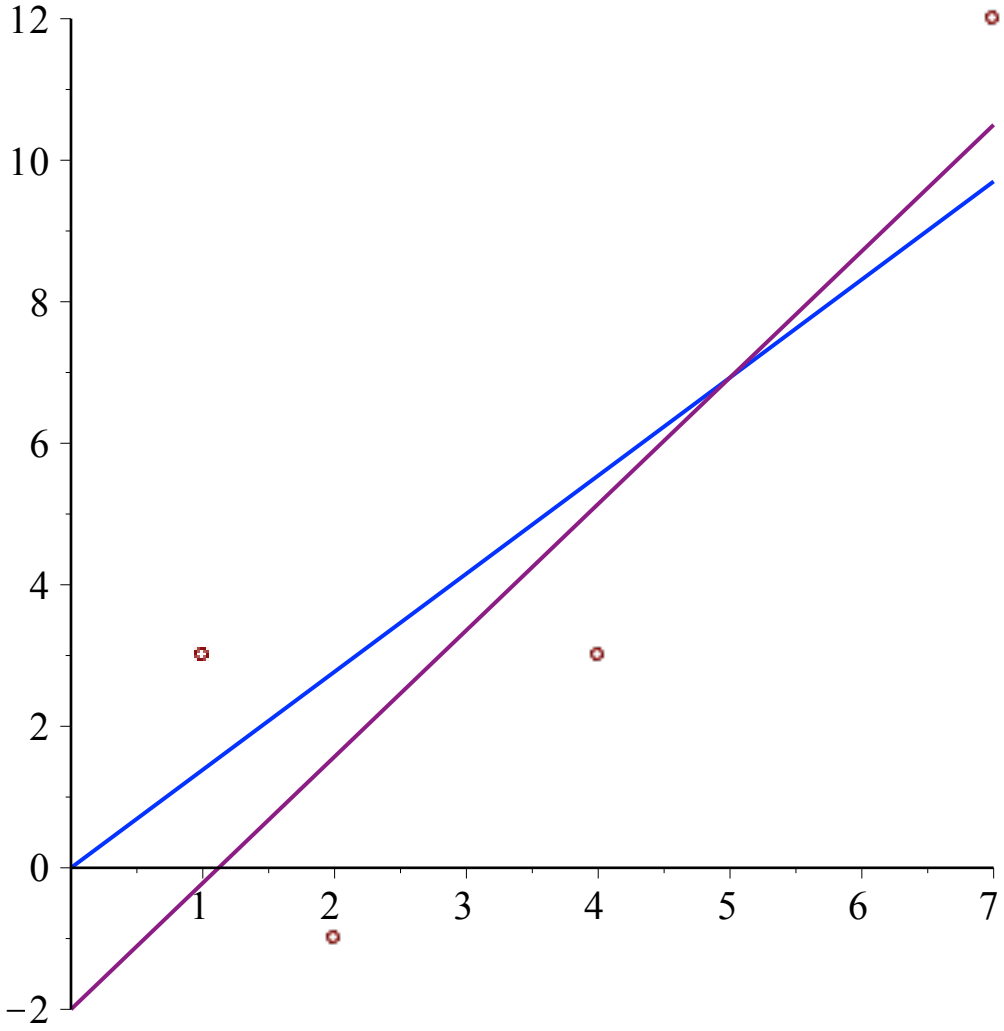
$f(x);$
$g(x);$

$$\frac{97}{70}\,x$$
$$\frac{25}{14}\,x - 2 \qquad \textbf{(6)}$$

$plot2 := plot(f, 0..7, colour = blue):$
$display([plot1, plot2]);$




```
plot3 := plot(g, 0..7, colour = purple) :
display([plot1, plot2, plot3]);
```




```
sumsqdev := proc(pts, f)
 local n, S, i :
 n := nops(pts) :
 S := 0 :
 for i from 1 to n do
   S := S + (f(pts[i][1]) − pts[i][2])^2 :
 od:
 evalf(S);
 end;
```

```
proc(pts, f)                                                                    (7)
    local n, S, i;
    n := nops(pts);
    S := 0;
    for i to n do S := S + (f(pts[i][1]) − pts[i][2])^2 end do;
    evalf(S)
end proc
```

$sumsqdev(pts, f);$
$sumsqdev(pts, g);$

$$28.58571429$$

$$23.78571429 \quad \textbf{(8)}$$

$evalf(f(x));$

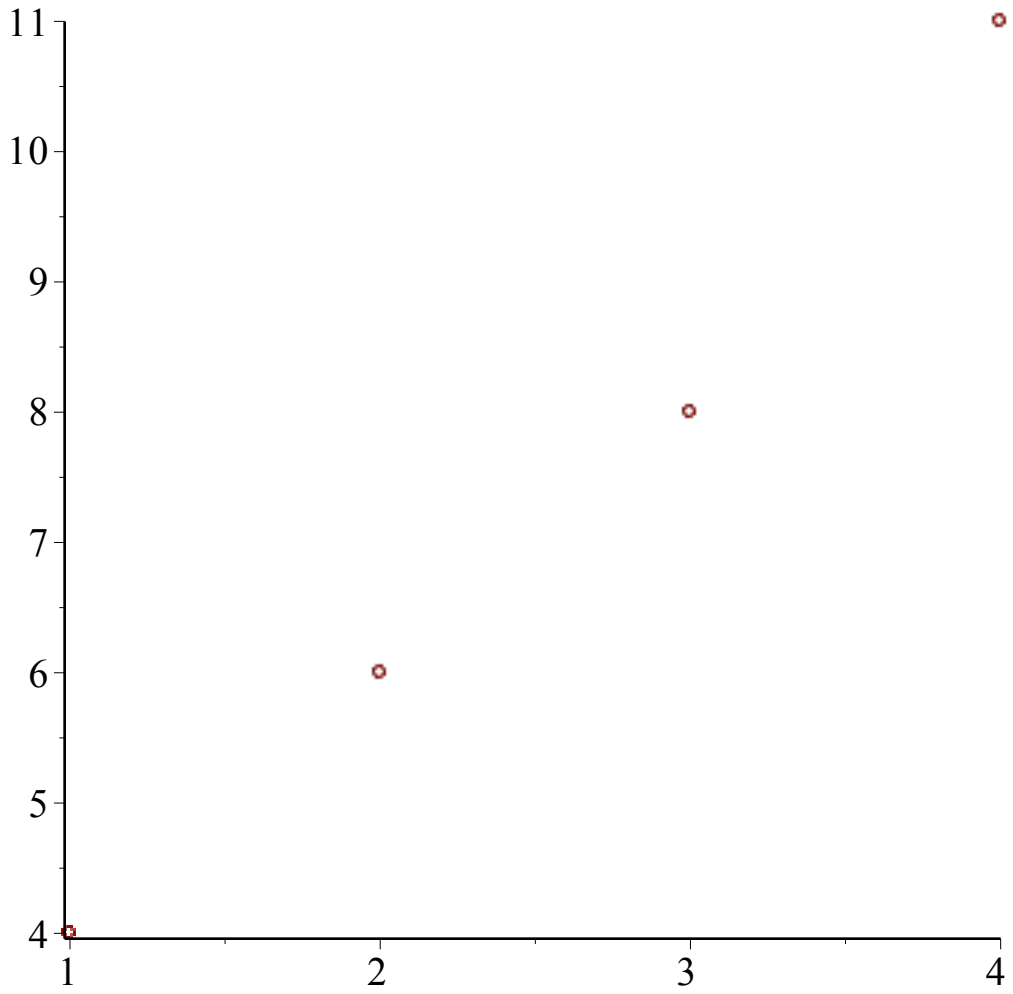
$$1.385714286\,x \quad \textbf{(9)}$$

$sumsqdev(pts, x \rightarrow 2 \cdot x - 2);$

$$27. \quad \textbf{(10)}$$

$pts := [[1, 4], [2, 6], [3, 8], [4, 11]];$
$plot1 := plot(pts, symbol = circle, style = point):$
$display(plot1);$

$$[[1, 4], [2, 6], [3, 8], [4, 11]]$$



$f := bestFitLineThroughOrigin(pts);$
$g := bestFitLine(pts);$

$$x \rightarrow C\,x$$

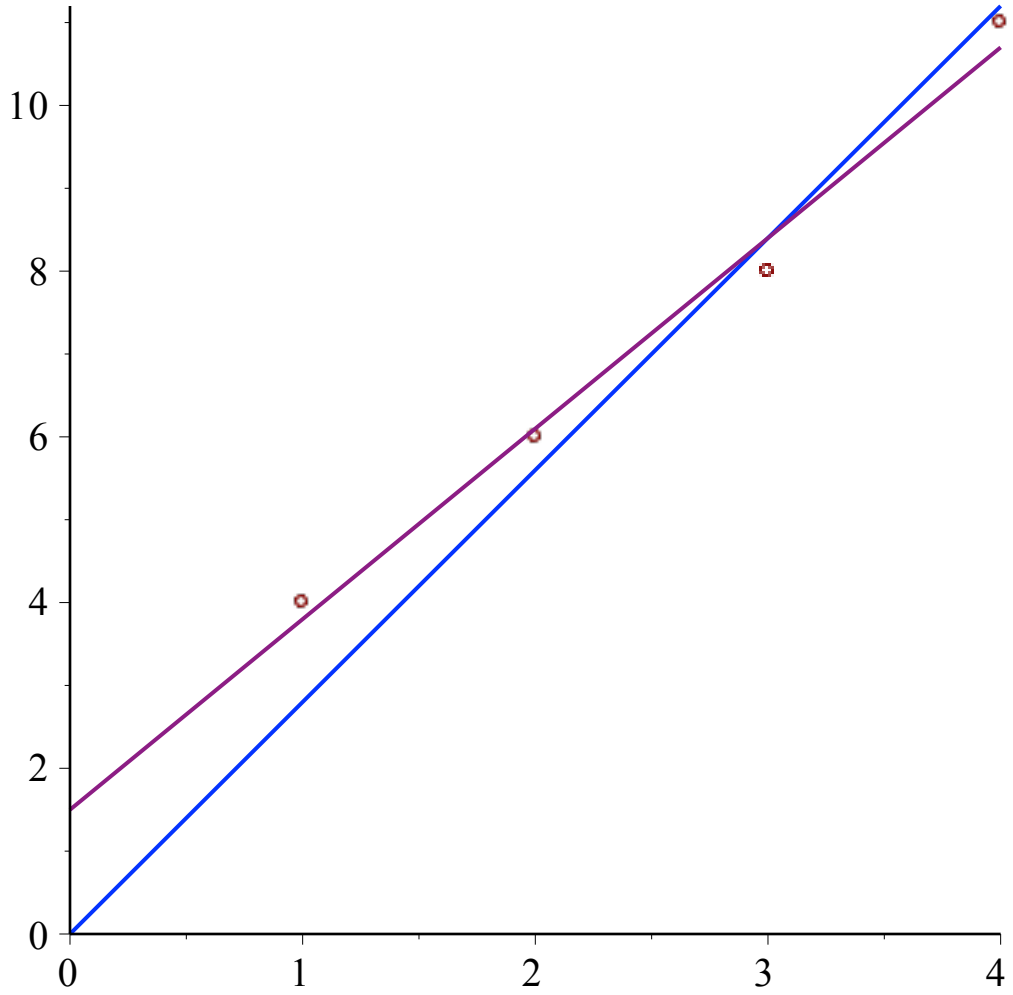
$$x \rightarrow A\,x + B \quad \textbf{(11)}$$

```
evalf(f(x));
evalf(g(x));
```

$$2.800000000\,x$$

$$2.300000000\,x + 1.500000000 \qquad \textbf{(12)}$$

```
plot2 := plot(f, 0..4, colour = blue):
plot3 := plot(g, 0..4, colour = purple):
display([plot1, plot2, plot3]);
```




```
sumsqdev(pts, f);
sumsqdev(pts, g);
```

$$1.800000000$$

$$0.3000000000 \qquad \textbf{(13)}$$

```
pts := [[1, 1.2], [2, 3.8], [3, 8], [4, 16.5]];
```

$$[[1, 1.2], [2, 3.8], [3, 8], [4, 16.5]] \qquad \textbf{(14)}$$

```
plot1 := plot(pts, symbol = circle, style = point):
bestFitCxSquared := proc(pts)
 local n, dn, nm, i, C:
 n := nops(pts):
```

```
dn := 0 :
nm := 0 :
for i from 1 to n do
nm := nm + pts[i][2]·(pts[i][1])^2 :
dn := dn + (pts[i][1])^4 :
od:
C := nm/dn :
x→C·x^2;
end
```

```
proc(pts)                                                                    (15)
    local n, dn, nm, i, C;
    n := nops(pts);
    dn := 0;
    nm := 0;
    for i to n do nm := nm + pts[i][2]*pts[i][1]^2; dn := dn + pts[i][1]^4 end do;
    C := nm/dn;
    x→C*x^2
end proc
```

```
f := bestFitCxSquared(pts);
g := bestFitLine(pts);
```

$$x \to C\,x^2$$

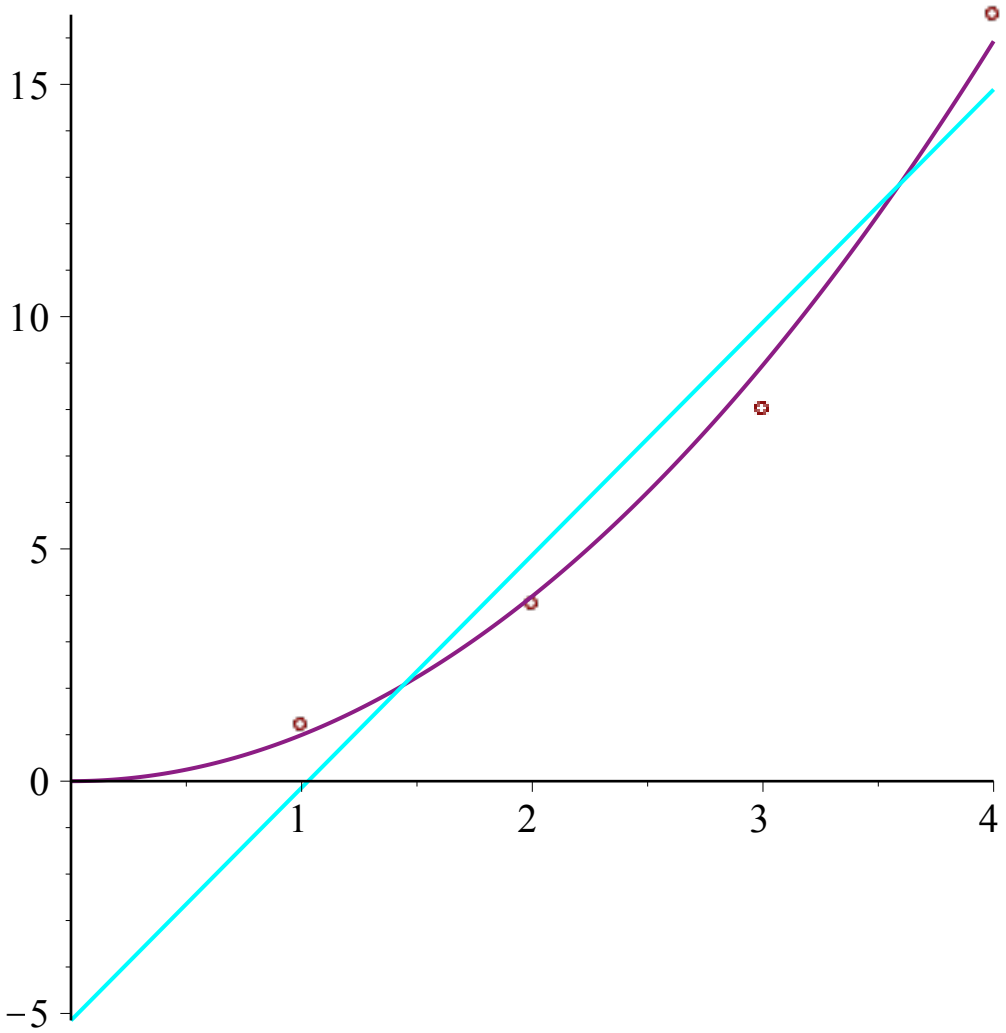
$$x \to A\,x + B \qquad (16)$$

```
evalf(f(x));
evalf(g(x));
```

$$0.9954802260\,x^2$$

$$5.010000000\,x - 5.150000000 \qquad (17)$$

```
plot2 := plot(f, 0..4, colour = purple) :
plot3 := plot(g, 0..4, colour = cyan) :
display([plot1, plot2, plot3]);
```




```
sumsqdev(pts, f);
```

1.322768357 **(18)**

```
sumsqdev(pts, g);
```

19.67600000 **(19)**

```
#routine to best fit a quadratic
#
bestFitQuadratic := proc(pts)
 local n, sumxfourths, sumxcubes, sumxsquares, sumxs, sumytimesxsquares, sumytimesxs, sumys, x, y, i,
      A, B, C, eq1, eq2, eq3 :
 n := nops(pts) :
 sumxfourths := 0 :
 sumxcubes := 0 :
 sumxsquares := 0 :
 sumxs := 0 :
 sumytimesxsquares := 0 :
 sumytimesxs := 0 :
 sumys := 0 :
 for i from 1 to n do
 x := pts[i][1] :
```

$y := pts[i][2]:$
$sumxfourths := sumxfourths + x^4:$
$sumxcubes := sumxcubes + x^3:$
$sumxsquares := sumxsquares + x^2:$
$sumxs := sumxs + x:$
$sumytimesxsquares := sumytimesxsquares + y \cdot x^2:$
$sumytimesxs := sumytimesxs + y \cdot x:$
$sumys := sumys + y:$
od:
$eq1 := sumxfourths \cdot A + sumxcubes \cdot B + sumxsquares \cdot C = sumytimesxsquares:$
$eq2 := sumxcubes \cdot A + sumxsquares \cdot B + sumxs \cdot C = sumytimesxs:$
$eq3 := sumxsquares \cdot A + sumxs \cdot B + n \cdot C = sumys:$
$print(eq1);$
$print(eq2);$
$print(eq3);$
$solve(\{eq1, eq2, eq3\}, \{A, B, C\});$
$subs(\%, x \rightarrow A \cdot x^2 + B \cdot x + C);$
end;

proc(*pts*) **(20)**

```
local n, sumxfourths, sumxcubes, sumxsquares, sumxs, sumytimesxsquares, sumytimesxs,
sumys, x, y, i, A, B, C, eq1, eq2, eq3;
n := nops(pts);
sumxfourths := 0;
sumxcubes := 0;
sumxsquares := 0;
sumxs := 0;
sumytimesxsquares := 0;
sumytimesxs := 0;
sumys := 0;
for i to n do
    x := pts[i][1];
    y := pts[i][2];
    sumxfourths := sumxfourths + x^4;
    sumxcubes := sumxcubes + x^3;
    sumxsquares := sumxsquares + x^2;
    sumxs := sumxs + x;
    sumytimesxsquares := sumytimesxsquares + y*x^2;
    sumytimesxs := sumytimesxs + y*x;
    sumys := sumys + y
end do;
```

```
    eq1 := sumxfourths * A + sumxcubes * B + sumxsquares * C = sumytimesxsquares;
    eq2 := sumxcubes * A + sumxsquares * B + sumxs * C = sumytimesxs;
    eq3 := sumxsquares * A + sumxs * B + n * C = sumys;
    print(eq1);
    print(eq2);
    print(eq3);
    solve({eq1, eq2, eq3}, {A, B, C});
    subs(%, x→A*x^2 + B*x + C)
end proc
pts := [[1, 1.2], [2, 3.8], [3, 8], [4, 16.5]];
```

$$[[1, 1.2], [2, 3.8], [3, 8], [4, 16.5]] \quad \textbf{(21)}$$

```
h := bestFitQuadratic(pts);
```

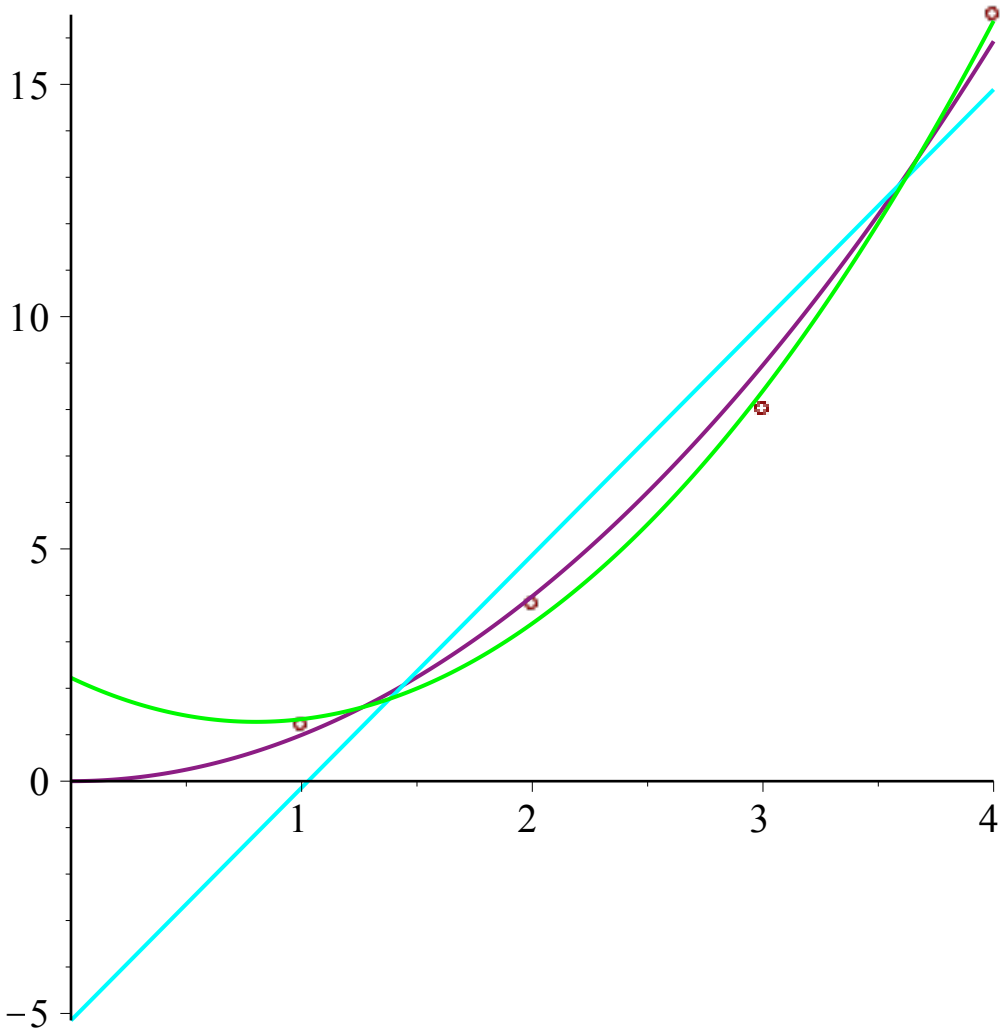
$$354\,A + 100\,B + 30\,C = 352.4$$

$$100\,A + 30\,B + 10\,C = 98.8$$

$$30\,A + 10\,B + 4\,C = 29.5$$

$$x \to 1.475000000\,x^2 + (-2.365000000)\,x + 2.225000000 \quad \textbf{(22)}$$

```
plot4 := plot(h, 0..4, colour = green):
display([plot1, plot2, plot3, plot4]);
```



$sumsqdev(pts, f);$

$sumsqdev(pts, h);$

1.322768357

0.3645000000 **(23)**

?*leastsquare*