# Mathematics, Physics and *A Hard Day's Night*

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**Abstract**

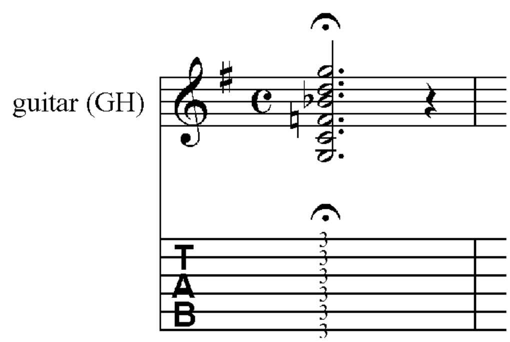
In this article we shall use mathematics and the physics of sound to unravel one of the mysteries of rock 'n' roll – how did the Beatles play the opening chord of *A Hard Day's Night*? The song may never sound the same to you again.

## 1 Introduction

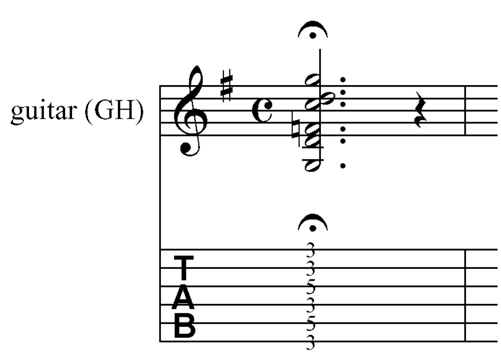
It was forty years ago that the Beatles ushered in a new era in pop music with the opening of *A Hard Day's Night*. The importance of the opening chord was clearly apparent to the Beatles. In *The Complete Beatles Recording Sessions* [3], author Mark Lewisohn quotes the Beatles' producer, George Martin, "We knew it would open both the film and the soundtrack LP, so we wanted a particularly strong and effective beginning. The strident guitar chord was the perfect launch." This seemed to close the discussion about the origin of the chord but should it have?

Many a guitarist (whether professional or amateur) has tried to reproduce the chord, but the voicing of the chord has remained a subject of much discussion over the past 40 years. If you browse musical transcriptions for the chord, you will come across three common ones (note that guitar parts are scored as usual an octave up from where they sound):

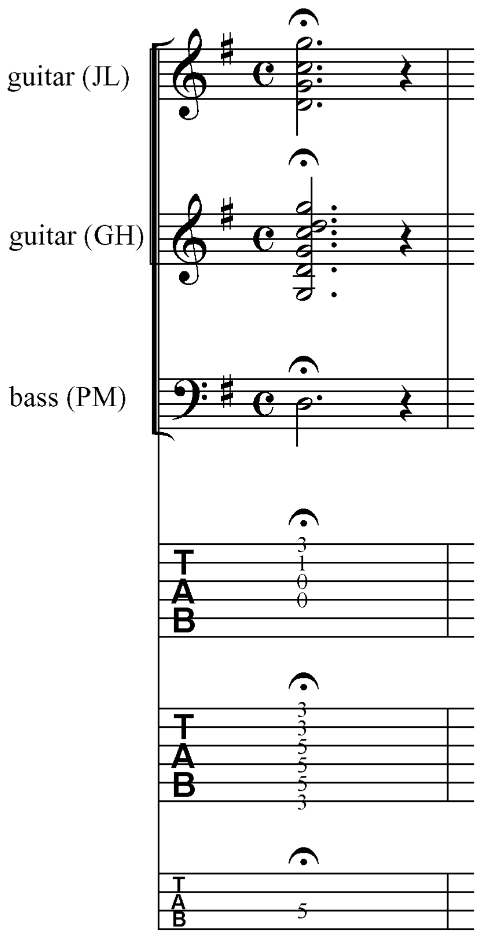
- Version 1: G C F Bb D G (a favorite by its ease of play; just play a barre chord at the third fret).




- Version 2: G D F C D G (another favorite, and one that often appears on Internet sites as the "one" that George Harrison (GH) played).



- Version 3: This one has George, John Lennon (JL) and Paul McCartney (PM) playing, with George (on his twelve string electric) playing G D G C D G, slightly different than in version 2, while John plays D G C G and Paul plays D on his bass (this is the transcription from [1]).




Is there any way to tell which of these three versions is the right one? Mathematics will help direct us to the answer.

## 2 Musical Forensics

We need to discuss briefly the physics and mathematics of sound (see [5, 2] for more details). Pure tones have a frequency, which corresponds to its pitch, and an amplitude, which corresponds roughly (but not exactly) to the loudness of the tone, and are modelled mathematically by sine and cosine functions. All sounds are made up of pure tones, which add together to give you complex tones and chords (that is, the functions of the latter are linear combinations of the functions for the pure tones). A single note sounded on an instrument is made up of a fundamental (main) pure tone plus other tones, called harmonics, whose frequencies are multiples of the fundamental tone's frequency. All of the amplitudes are added together in this "mix" that we hear. When a sound is digitized for a CD, the amplitude is sampled 44,100 times every second. What is hidden in the process of recording music are the individual frequencies, and how they were played. Fourier Transforms can be used to dissassemble the sampled amplitudes into the original frequencies.

After the song *A Hard Day's Night* was opened in a sound editing program on a computer, a segment of approximately one second was selected in the middle of the chord. The sound was saved as a file, and using some Mathematica subroutines from [2, chapter 14] a Fourier Transform was run on the list of data. There were 29,375 frequencies present, which included not only the notes being struck, but also harmonics, as well as any other frequencies that might have arisen during the recording.

We are after the loudest notes, as these correspond to the fundamental notes being struck (though there will probably be some of the louder harmonics present, along with possibly some other loud rattles). A threshhold was chosen which kept the sound faithful to the original. The table shows the 48 frequencies with amplitude 0.02 or larger.

| Freq. (Hz) | Ampl. | Freq. (Hz) | Ampl. | Freq. (Hz) | Ampl. | Freq. (Hz) | Ampl. |
|---|---|---|---|---|---|---|---|
| 110.34 | 0.0600967 | 299.494 | 0.0298296 | 1050.86 | 0.0687151 | 2368.93 | 0.0221358 |
| 145.619 | 0.025485 | 392.57 | 0.0309716 | 1185.97 | 0.0372155 | 2371.19 | 0.0212846 |
| 148.621 | 0.0264278 | 438.358 | 0.0286329 | 1286.55 | 0.0231789 | 2371.94 | 0.0436633 |
| 149.372 | 0.0656018 | 524.678 | 0.0680974 | 1314.32 | 0.03819 | 2372.69 | 0.036042 |
| 150.123 | 0.175149 | 587.73 | 0.020613 | 1320.33 | 0.0223535 | 2637.65 | 0.0261839 |
| 174.142 | 0.0275547 | 588.48 | 0.0310337 | 1321.08 | 0.0494908 | 2638.4 | 0.0237794 |
| 174.893 | 0.0380282 | 589.231 | 0.0231753 | 1488.47 | 0.0241328 | 2754. | 0.020001 |
| 175.643 | 0.0407103 | 785.141 | 0.0323532 | 1632.58 | 0.0205742 | 2763.76 | 0.0493617 |
| 195.159 | 0.0405164 | 786.642 | 0.0251928 | 1750.43 | 0.0234704 | 3083.52 | 0.0332062 |
| 218.428 | 0.0448308 | 787.393 | 0.0268553 | 2359.93 | 0.0366079 | 3147.32 | 0.0293723 |
| 261.964 | 0.0302402 | 960.784 | 0.0228509 | 2367.43 | 0.0267098 | 3148.07 | 0.0418507 |
| 262.714 | 0.0234502 | 981.801 | 0.02242 | 2368.18 | 0.0755327 | 3158.58 | 0.0285631 |

The frequencies need to be converted to notes, so choosing the reference note A 220 Hz, the frequencies were converted to the number of semitones above or below A 220 (by applying the function $f(x) = 12\log_2(x/220)$). Here is the list of semitones (we see that some of the instruments could have been better tuned as not all of the numbers are close to their nearest integer):

$-11.9466$, $-7.14367$, $-6.79035$, $-6.70313$, $-6.61635$, $-4.04686$, $-3.97239$, $-3.89825$, $-2.07421$, $-0.124124$, 3.02237, 3.07191, 5.34031, 10.0254, 11.9353, 15.0472, 17.0118, 17.0339, 17.056, 22.0254, 22.0584, 22.075, 25.5205, 25.8951, 27.0719, 29.1659, 30.5752, 30.9449, 31.0238, 31.0337, 33.099, 34.699, 35.9056, 41.078, 41.133, 41.1385, 41.1439, 41.1604, 41.1659, 41.1714, 43.0042, 43.0091, 43.7514, 43.8127, 45.708, 46.0626, 46.0667, 46.1244

In musical circles, middle C is written as C4, with the second number indicating the octave, so A 220 Hz is written as A3. Here are the frequencies above rounded to the nearest semitones:

A2, D3, D3, D3, D3, F3, F3, F3, G3, A3, C4, C4, D4, G4, A4, C5, D5, D5, D5, G5, G5, G5, B5, B5, C6, D6, E6, E6, E6, E6, F#6, G#6, A6, D7, D7, D7, D7, D7, D7, D7, E7, E7, F7, F7, G7, G7, G7, G7

Many of the notes appear in the various versions of "the chord". But to argue what was played and how the notes were played we'll need to make some deductions.

Some of the notes (especially in the higher range) must be harmonics, as they are well beyond what instruments can play. In fact, the range of a guitar is from E2 to about E6 and the bass guitar from E1 to about D4. Notes could have arisen on either George Harrison's or John Lennon's guitar or Paul's bass. The analysis now shows why the three well known transcriptions of the opening chord must all be wrong: each has a low G2 being played, but this note is definitely missing.

It is well known that for the album *A Hard Day's Night*, George used a 12 string guitar and its sound can definitely be heard on the solo in *A Hard Day's Night*. Thus it seems safe to assume George used this guitar on the opening chord as well. The twelve string guitar has each string doubled, with the bottom four in octaves, so the strings are, from lowest to highest, E2 E3 A2 A3 D3 D4 G3 G4 B3 B3 E4 E4. It seems reasonable that notes on strings of roughly the same thickness struck on one instrument would be roughly of the same amplitude. Looking back at the frequencies and their amplitudes, we see that one D3 is extra loud, with an amplitude of 0.175. This is taken as a bass note from Paul's Hofner bass (no other single frequency is nearly as loud).

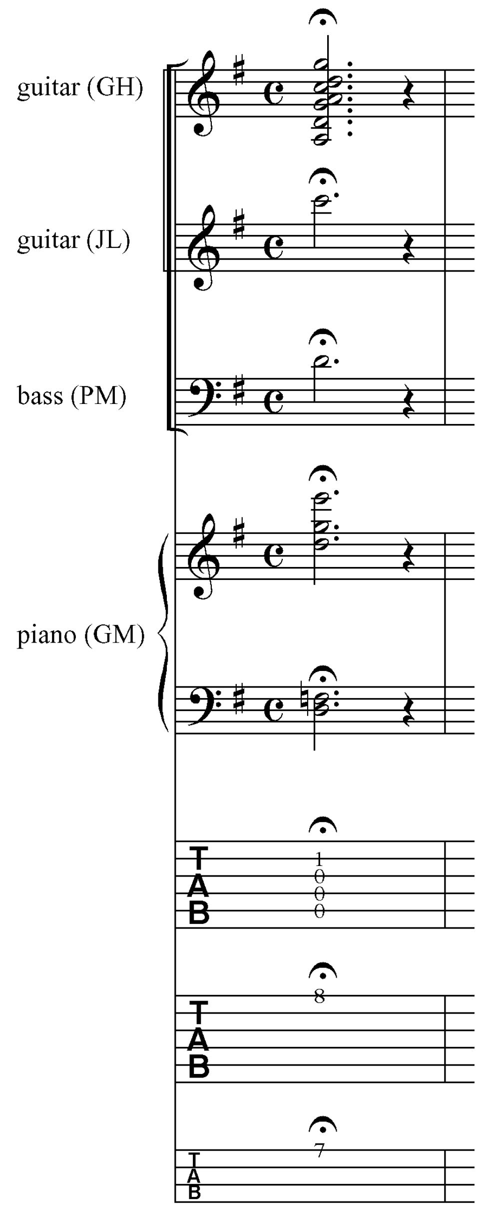
Now A2 and A3 can be paired off, both likely coming from George's 12 string (a nice open pair of strings). But even with one of the D3's accounted for on Paul's bass, what about the other three D3's?

Only one can come from George's 12 string, and even if John played another one on his six string, there's still another to account for. There is no evidence that any guitar was multitracked, at least on this opening chord. The two F3s create a much bigger problem. For no matter how George plays an F3 on his 12 string, an F4 should be heard as well, and there is no F4 at all present!

A hidden assumption came to the fore. Beatles' record producer, George Martin, is known to have doubled on piano George Harrison's solo on the track. Could "the chord" be part piano? Pianos have three strings for every note; a hammer strikes all three at the same time to produce a sound. That solved the problem of the three F3's: all could have come from a piano playing F3. Note that the frequencies of the three F3s were slightly different, but each string on a piano is individually tuned and is likely to be slightly off from one another in the "triple."

But what about the three left over D3's? If all belonged to a single piano note, then where would the single D4 come from? Not from George Harrison's guitar (as a D3 or another D4 would be present) and not from George Martin's piano (as otherwise there would be three D4's present). However, the bottom ten pitches on a piano are single strings which change to pairs of strings, and around C3 they change to the usual triples of strings. But indeed there are some grand pianos (of medium length) for which the break occurs right after D3. This implied that two of the D3s were played on the piano.

What George Harrison played on his 12 string was nothing like any of the transcriptions: he played A2 A3 D3 D4 G3 G4 C4 C4, most likely on string sets 2 through 5 – eight strings with six open strings in total; (for a great chiming effect). George Martin (GM) played D3 F3 D5 G5 E6 on the piano. The other notes are fairly high and could be attributed to harmonics of these notes, except that there is a loud C5, which could have been played by John high up on his six string. There is also one extra E6 unaccounted for, which is taken as a harmonic.




## 3 The End

The notes played on the piano interweave well with the notes on the 12 string, starting a bit higher (at

D3) from the lowest note played on the guitar and ending higher (at E6). The amplitudes show why the piano is so well hidden; it is mixed perfectly, with amplitudes almost identical to those of the higher strings played on Harrison's guitar.

In *All You Need is Ears* [4], George Martin makes a point of saying "it shouldn't be expected that people are necessarily doing what they appear to be doing on records" and likens recording to filmmaking, where all sorts of effects are carried out in the background in order to create illusions. We see that sometimes mathematics can unravel the best mysteries.


### Acknowledgements

This article was partially supported by a grant from the Natural Sciences and Engineering Research Council of Canada.


## References


[1] T. Fujita, Y. Hagino, H. Kubo and G. Sato, *The Beatles Complete Scores*, Hal Leonard, Milwaukee, 1993.

[2] T.W. Gray and J. Glynn, *Exploring Mathematics with Mathematica*, Addison Wesley, New York, 1991.

[3] M. Lewisohn, *The Complete Beatles Recording Sessions*, Doubleday, Toronto, 1988.

[4] G. Martin, *All You Need Is Ears*, St. Martins' Press, New York, 1979.

[5] J.S. Rigden, *Physics and the Sound of Music*, Wiley, New York, 1977, pg. 71.